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A NUMERICAL STUDY OF AN IDEALIZED EMP PROBLEM.(U)

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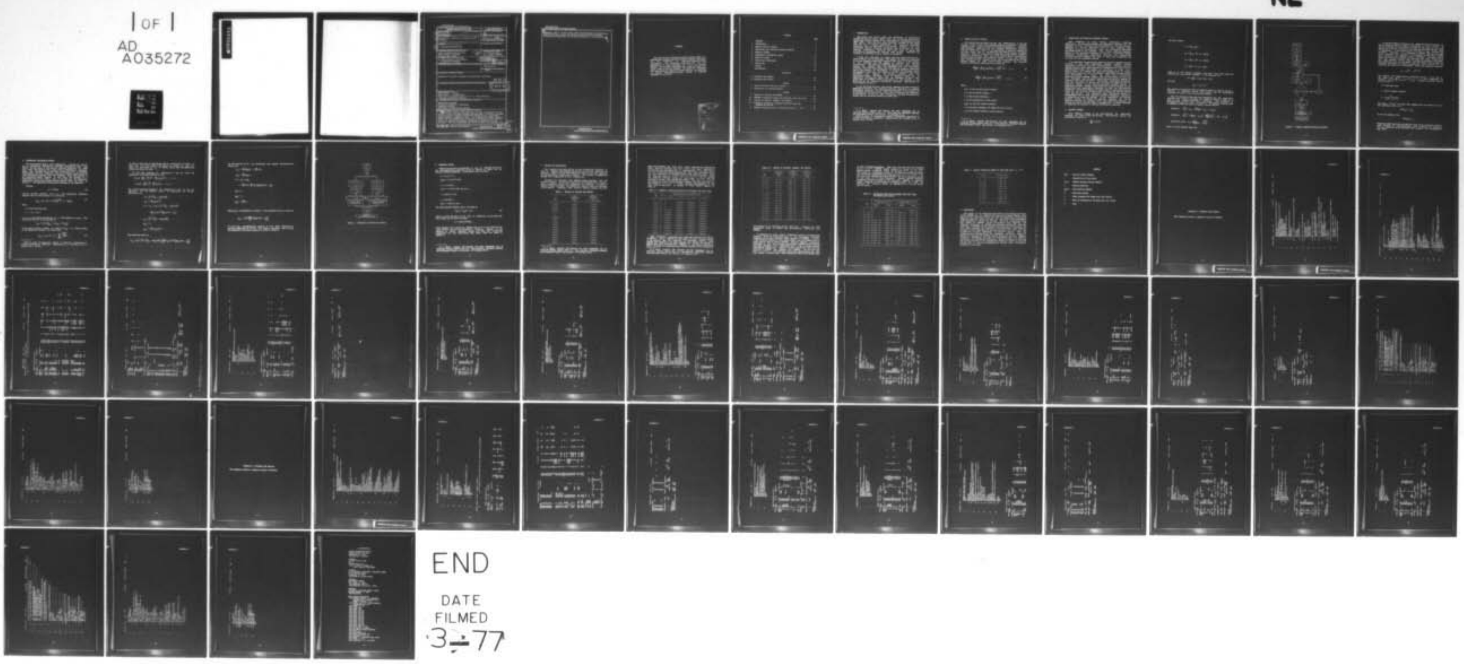

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Accurate results were obtained for the time history of the electric-field strength between two parallel, infinite, aluminum plates caused by Compton electrons generated by a transient, high-intensity, gamma-ray flux. Two numerical methods, Hamming's method and an exponential differencing technique, were used to solve the resulting ordinary differential equations of the problem. The two techniques and their results are examined and over

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compared, and it is set forth that the exponential differencing method provides a more efficient solution to the ordinary differential equations of the type involved in this problem.

FOREWORD

The Army Multiple Systems Evaluation Program (MSEP) is a comprehensive program developing general analytic techniques for the prediction of high-electromagnetic-pulse vulnerability and hardening technology and for the application of these techniques to a list of critical systems. The analytic techniques have been verified for a large class of tactical systems. The hardening techniques have been applied to specific systems and are now resulting in product improvement programs leading to hardened equipment in the field.

CONTENTS

APPENDICES

FIGURES

TABLES

1. INTRODUCTION

This effort was written under the sponsorship of the Multiple Systems Evaluation Program (MSEP) which has as its main objective to harden Army tactical systems to the exoatmospheric electromagnetic-pulse (EMP) threat. Along with this major objective, the MSEP is also tasked with the aim to develop experimental and analytical evaluation techniques that are applicable to all systems' problems. This work illuminates several numerical techniques that can be used to solve ordinary differential equations that might arise in making EMP vulnerability assessments for tactical Army systems. Therefore, the objective of providing analytic methods is satisfied.

A study was undertaken to expand upon the results obtained in a paper by Wyatt[1] and to investigate and compare two numerical techniques in solving ordinary differential equations that occur in an EMP problem. It was conducted with the purpose of refining the previous calculations.[1] The numerical techniques employed are fairly standard methods, but whereas Wyatt was concerned only with approximate solutions, this work deals mainly with the specific numerical techniques used in the solution of the problem. An analysis of truncation errors was employed to deal with the accuracy of the solutions that better solidifies the results. The methods utilized were the predictor-corrector technique known as Hamming's method and an exponential differencing method that was developed by Pope.[2] This effort does not attempt to explain the physics of the problem or the derivation of the ordinary differential equations. This material is fully detailed by Wyatt.[1]

In the following sections, some general comments are made concerning Runge-Kutta solutions and predictor-corrector solutions of ordinary differential equations. The two techniques employed in the solution of the problem are described fully, along with a discussion and comparison of the results. At the end, some conclusions are made concerning the results and the numerical methods that were used.

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[1] *W. T. Wyatt, Internal EMP Strength and Time Dependence for an Idealized Problem, Report 1994, U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, VA (February 1971).*

[2] *David A. Pope, An Exponential Method of Numerical Integration of Ordinary Differential Equations, Communications of the ACM 6, No. 8 (August 1963), 491-493.*

2. GENERAL PHYSICAL PROBLEM

The problem that was dealt with was the determination of the time history of the electric-field strength between two parallel, infinite, aluminum plates caused by Compton electrons generated by a transient, high-intensity, gamma-ray flux. The ordinary differential equations that are solved are a result of the application of Poisson's equation to the free charge that is the spatially distributed current of Compton electrons. Since the conductivity of the air is significant, Poisson's equation is modified by Ohm's law, which relates the air conductivity, electric-field strength, and conduction current. In the case examined here, methane was used instead of air, and it was found by Wyatt[1] that methane reduced the peak electric-field strength. The resulting ordinary differential equations to be solved are

$$\frac{dE(t)}{dt} + \frac{q\mu}{\varepsilon} N_e(0)e^{rt}E(t) = \frac{J_c(t)}{\varepsilon} e^{rt} , \quad t < 0 \tag{1}$$

$$\frac{dE(t)}{dt} + \frac{q\mu}{\varepsilon} N_e(t)E(t) = \frac{J_c(t)}{\varepsilon} , \quad t \geq 0 , \tag{2}$$

where

$E(t)$ is the electric-field strength,

q is the electronic charge,

μ is the electron mobility,

ε is the permittivity of free space,

$N_e(t)$ is the free electron density,

r is the model parameter for gamma-flux rate history,

J_c is the Compton electron current density.

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[1] *W. T. Wyatt, Internal EMP Strength and Time Dependence for an Idealized Problem, Report 1994, U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, VA (February 1971).*

3. RUNGE-KUTTA AND PREDICTOR-CORRECTOR METHODS

When attempting to solve ordinary differential equations numerically, two well-known methods, the Runge-Kutta technique and the predictor-corrector technique, are often employed to find the solution. Depending on various constraints such as time, computing funds available, accuracy, and stability, there are accompanying advantages and disadvantages to both methods. Thus, before examining Hamming's method in the solution of equations (1) and (2), a few general remarks concerning fourth-order Runge-Kutta solutions and fourth-order predictor-corrector methods are made to give some background and rationale for the technique utilized.

It is generally recognized that predictor-corrector methods are more difficult to code than Runge-Kutta methods. However, although Runge-Kutta techniques are more straightforward, predictor-corrector methods provide a much easier analysis and examination of errors and are generally much faster. For example, the evaluation of $f(x,y)$ (i.e., $dy/dx = f(x,y)$) is usually the most time-consuming part of solving differential equations, and fourth-order Runge-Kutta methods require four evaluations of $f(x,y)$ per step, while fourth-order predictor-corrector methods require only two evaluations of $f(x,y)$ per step. This means that the fourth-order predictor-corrector methods are generally nearly twice as fast as fourth-order Runge-Kutta techniques. Thus, the evaluation of errors and the speed of computation are two compelling reasons to use predictor-corrector methods instead of Runge-Kutta methods. However, since predictor-corrector methods are not self-starting and Runge-Kutta techniques do have the self-starting capability, Runge-Kutta methods are quite useful in generating starting values for the solution and may be used to change the interval between steps when desired. This usefulness makes Runge-Kutta methods an indispensable tool in using predictor-corrector techniques. Therefore, a combination of these two methods, (1) the Runge-Kutta to determine starting values and change the per-step interval and (2) the predictor-corrector to actually solve the differential equation and analyze the errors involved, gives a technique that is highly desirable in computing the solution of ordinary differential equations.

4. HAMMING'S METHOD

Since Hamming's method is not self-starting, the Runge-Kutta technique was used to start the solution. This involves writing the differential equation as

$$\frac{dy}{dx} = f(x,y)$$

and then finding

$$k_1 = f\left(x_i, y_i\right)h ,$$

$$k_2 = f\left(x_i + \tfrac{1}{2}h,\ y_i + \tfrac{1}{2}k_1\right)h ,$$

$$k_3 = f\left(x_i + \tfrac{1}{2}h,\ y_i + \tfrac{1}{2}k_2\right)h ,$$

$$k_4 = f\left(x_i + h,\ y_i + k_3\right)h ,$$

where h is the desired increment step size. Once these values are computed for a particular increment, Δy is calculated by

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) ,$$

and then

$$y_{i+1} = y_i + \Delta y .$$

This process is continued until the desired number of starting values is obtained. In Hamming's method, three values from the Runge-Kutta technique are used to start the solution process.

Once the starting values have been determined from the Runge-Kutta technique, the general procedure for Hamming's method is used, as is done by most predictor-corrector techniques. This procedure is illustrated by the flow chart in figure 1. The specific formulas used in Hamming's method and shown in figure 1 are as follows:

Predictor: $$y_{i+1}^{(0)} = y_{i-3} + \frac{4h}{3}\left(2y_i' - y_{i-1}' + 2y_{i-2}'\right),$$

Corrector: $$y_{i+1}^{(j+1)} = \frac{1}{8}\left(9y_i - y_{i-2}\right) + \frac{3h}{8}\left(\left[y_{i+1}^{(j)}\right]' + 2y_i' - y_{i-1}'\right),$$

Truncation error: $$T_i \simeq \frac{9}{121}\left[y_{i+1} - y_{i+1}^{(0)}\right] ,$$

where h is the interval step size.

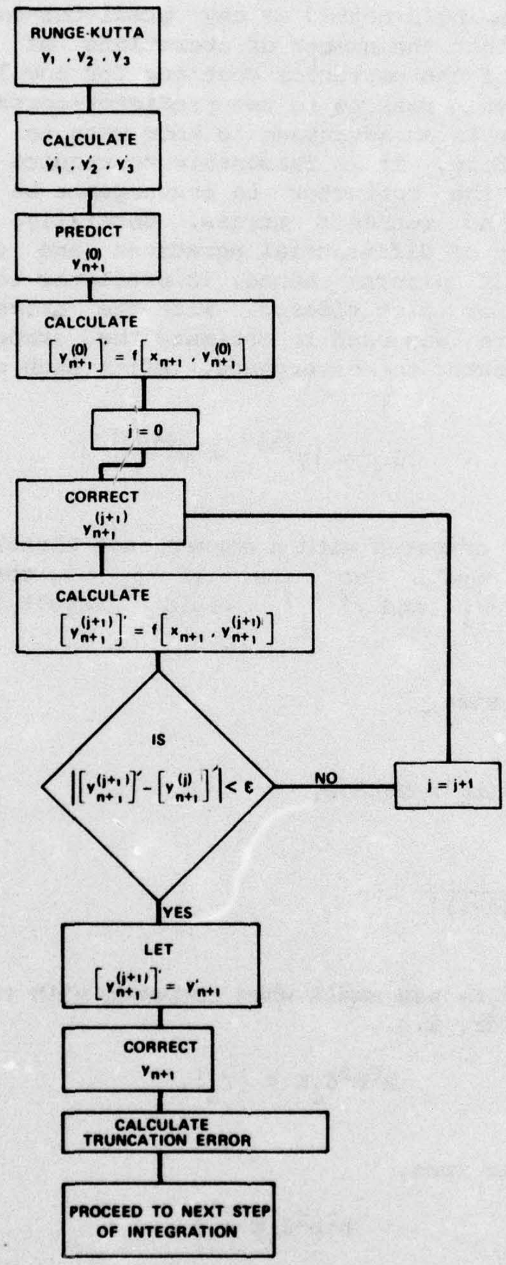

Figure 1. General Predictor-Corrector Method.

In applying Hamming's method or any predictor-corrector method, one must be cautious about the number of iterations of the corrector. To let the iteration of the corrector continue for any length of time would defeat one of the main reasons to use predictor-corrector methods, speed of computation. It is an advantage to know when to stop iterating the corrector. Therefore, it is reasonable to require that any error made by not iterating the corrector to convergence be small when compared with truncation and roundoff errors. Generally, truncation errors affect the solution of differential equations and cause instabilities more than roundoff errors; hence, in predictor-corrector techniques, only truncation error is considered. With the above point in mind, the following procedure was used to estimate the error incurred by not iterating the corrector to convergence. After each corrector iteration,

$$\delta_i = |y^{(i)'} - y^{(i-1)'}|$$

was computed and compared with a convergence factor, ε (ε in fig. 1). The value ε was chosen so that if $\delta_i < \varepsilon$, then terminating the iteration with $\left[y^{(i)}\right]'$ and $y^{(i+1)}$ would result in an error value of $h^2 b^2 \delta_i K$, where

h = time step size,

$b = \frac{3}{8}$ for Hamming's method,

$$K = \frac{\delta_i}{|y^{(i)} - y^{(i-1)}|} .$$

This value, $h^2 b^2 \delta_i K$, was small when compared with the absolute value of the truncation error, i.e.,

$$h^2 b^2 \delta_i K < |T_n| .$$

In all the computer runs,

$$h^2 b^2 \delta_i K = 0 ,$$

which was indeed less than the absolute value of the truncation error in every instance of the calculations. Also, utilizing this procedure, only two iterations of the corrector were required for convergence in most cases.

5. EXPONENTIAL DIFFERENCING METHOD

The second method used to solve differential equations (1) and (2) was an exponential differencing technique developed by Pope.[2] This method has been shown to have superior stability properties for large step sizes when dealing with a large class of differential equations. Hence, this technique may be used with a large step size to decrease significantly the total computing time for a solution, particularly in those engineering problems, like EMP problems, where high accuracy is not necessarily needed. However, in this work, the accuracy is a significant part of the effort, and, as it turns out, the exponential differencing method does provide precise results, which are shown in section 7. The exponential differencing technique is now described.

Consider

$$y' = f(x,y) \tag{3}$$

with the initial condition $y(x_o) = y_o$. The exponential difference equation used to solve equation (3) is given by

$$y_{n+1} = y_n + hf + y''f_y^{-2}\left(e^{hf_y} - 1 - hf_y\right), \tag{4}$$

where

h is the time step size,

$y'' = f_x + f \cdot f_y$,

f_x, f_y are the partial derivatives of f with respect to x and y. The truncation error for this algorithm is

$$T_{n+1} \approx \frac{1}{6} h^3 \left(f_{xx} + 2f \cdot f_{xy} + f^2 \cdot f_{yy}\right).$$

If the value of $|hf_y|$ is small, at least if $|hf_y| < 0.1$, then in place of the exponential formula, the series form

$$y_{n+1} = y_n + hf + y'' \sum_{k=2}^{\infty} \frac{h^k f_y^{k-2}}{k!} \tag{5}$$

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[2] *David A. Pope, An Exponential Method of Numerical Integration of Ordinary Differential Equations, Communications of the ACM, 6, No. 8 (August 1963), 491-493.*

is used to avoid loss of significance due to cancellation of terms. In this case, only a few terms of the series are needed; we used only three. If the value of hf_y is fairly large, then the exponential subroutines should be used.

To find the solutions for equations (1) and (2) using the exponential differencing method, we let

$$f(t,E) = \frac{dE}{dt} = \frac{J_c}{\varepsilon} e^{rt} - \frac{q\mu}{\varepsilon} N_e(0) e^{rt} E \ , \quad t < 0 \ ,$$

$$g(t,E) = \frac{dE}{dt} = \frac{J_c}{\varepsilon} - \frac{q\mu}{\varepsilon} N_e(t) E \ , \quad t > 0 \ .$$

The various equations needed to use equations (4) and (5) are now generated. For the function $f(t,E)$, its derivatives and partial derivatives are

$$f_t = \frac{r}{\varepsilon} e^{rt}\left[J_c - q\mu N_e(0)E\right] ,$$

$$f_E = - \frac{q\mu}{\varepsilon} N_e(0) e^{rt} ,$$

$$E'' = f_t + f \cdot f_E = \frac{r}{\varepsilon} e^{rt}\left[J_c - q\mu N_e(0)E\right]$$

$$+ \frac{q\mu}{\varepsilon^2} N_e(0) e^{2rt}\left[q\mu N_e(0)E - J_c\right] ,$$

$$f_{tt} = \frac{r^2}{\varepsilon} e^{rt}\left[J_c - q\mu N_e(0)E\right] ,$$

$$f_{EE} = 0 ,$$

$$f_{tE} = - \frac{q\mu}{\varepsilon} N_e(0) r e^{rt} .$$

The truncation error is

$$T_{n+1} \approx \frac{1}{6} h^3 \frac{r^2}{\varepsilon} e^{rt}\left[J_c - q\mu N_e(0)E\right] + \left\{\frac{2q\mu}{\varepsilon^2} N_e(0) e^{2rt}\left[q\mu N_e(0)E - J_c\right]\right\} .$$

For the function g(t,E), its derivatives and partial derivatives are found to be

$$g_t = -\frac{q\mu}{\varepsilon} EN_e'(t) = -\frac{q\mu}{\varepsilon} S \cdot E ,$$

$$g_E = -\frac{q\mu}{\varepsilon} N_e(t) ,$$

$$E'' = g_t + g \cdot g_E$$

$$= -\frac{q\mu}{\varepsilon} S \cdot E + \frac{q\mu}{\varepsilon} N_e(t) \left[q\mu N_e(t)E - J_c \right] ,$$

$$g_{tt} = 0 ,$$

$$g_{EE} = 0 ,$$

$$g_{tE} = -\frac{q\mu}{\varepsilon} S ,$$

where $N_e(t)$ is described in section 6. The truncation error is given by

$$T_{n+1} \approx \frac{1}{6} h^3 \left\{ \frac{2q\mu}{\varepsilon^2} \left[q\mu N_e(t)E - J_c \right] \right\} .$$

It is an easy, straightforward process to code these quantities to calculate equations (4) and (5). The general procedure used to solve the exponential difference equation is given in figure 2.

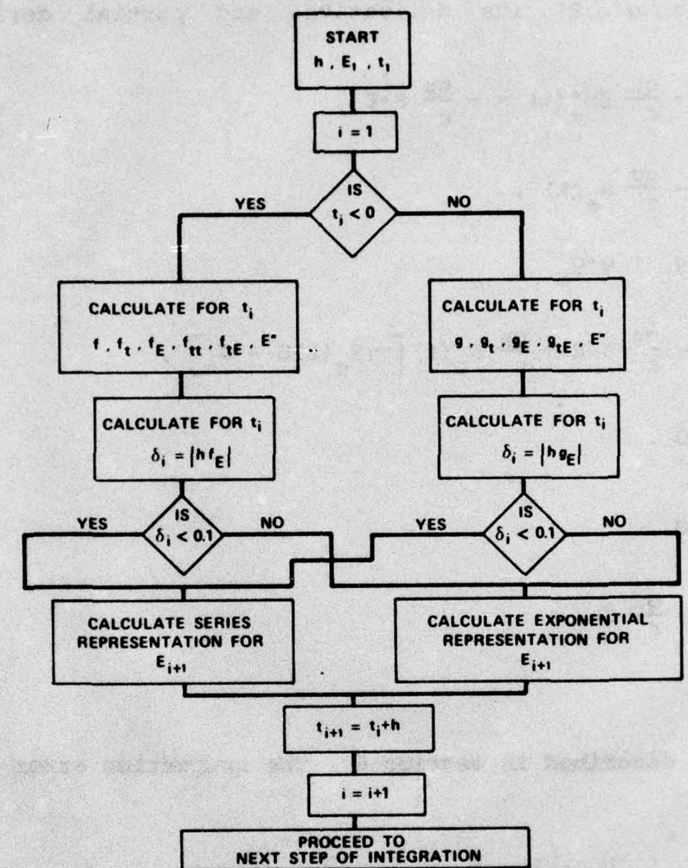

Figure 2. Exponential Differencing Method.

6. NUMERICAL VALUES

Finding the solution of equations (1) and (2) required the use of the same hypothetical numerical values as applied by Wyatt[1] for the various constant parameters. These values are as follows:

$q = 1.6 \cdot 10^{-19}$ C,

$N_e(0) = 1.133 \cdot 10^{11}/cm^3$,

$\mu = 2.0\ m^2/V/s$,

$J_c(t) = 1.34 \cdot 10^3\ A/m^2$ for all t ,

$\epsilon = 8.85 \cdot 10^{-2}$ F/m ,

$r = 5.0 \cdot 10^8/s$,

$E(0) = 3.697 \cdot 10^4$ V/m .

The free-electron density, $N_e(t)$, is given by

$$N_e(t) = N_e(0) + St , \tag{6}$$

where t is the time and S is the rate of production of ion pairs per unit volume and is given the value

$$S = 5.664 \cdot 10^{25}/m^3/s .$$

Since equation (6) is valid for methane only to 9 ns, any solutions for times beyond this were not possible to calculate. However, it was possible to obtain excellent results and make some interesting comparisons for this restrictive time frame, which are shown in section 7.

[1] *W. T. Wyatt, Internal EMP Strength and Time Dependence for an Idealized Problem, Report 1994, U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, VA (February 1971).*

7. RESULTS AND COMPARISONS

Two computer codes were written to solve differential equations (1) and (2). Computer code HAMMING was written to solve the equations by Hamming's method, and the code EXPDIFF was used to solve the equations by the exponential differencing methods. The listings of HAMMING and EXPDIFF are in appendices A and B, respectively.

Although the solutions from HAMMING compare favorably with the results of Wyatt,[1] the solutions obtained are considered more accurate, since truncation errors are calculated and examined. In table I, specific values from Wyatt[1] and HAMMING are listed to show the agreement of the results. Regarding these results, numerous computer runs were

TABLE I. RESULTS OF ORIGINAL AND HAMMING

Time (s)	Original Electric field strength (V/m)	HAMMING Electric field strength ($\Delta t = 1.0 \cdot 10^{-12}$) (V/m)
$-4.0.10^{-9}$	$3.80 \cdot 10^{4}$	$3.6970 \cdot 10^{4}$
$-3.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6965 \cdot 10^{4}$
$-2.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6961 \cdot 10^{4}$
$-1.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6960 \cdot 10^{4}$
0	$3.80 \cdot 10^{4}$	$3.6959 \cdot 10^{4}$
$5.0 \cdot 10^{-10}$	$3.00 \cdot 10^{4}$	$3.2129 \cdot 10^{4}$
$1.0 \cdot 10^{-9}$	$2.48 \cdot 10^{4}$	$2.6259 \cdot 10^{4}$
$1.5 \cdot 10^{-9}$	$2.10 \cdot 10^{4}$	$2.2092 \cdot 10^{4}$
$2.0 \cdot 10^{-9}$	$1.85 \cdot 10^{4}$	$1.9107 \cdot 10^{4}$
$3.0 \cdot 10^{-9}$	$1.47 \cdot 10^{4}$	$1.5093 \cdot 10^{4}$
$4.0 \cdot 10^{-9}$	$1.20 \cdot 10^{4}$	$1.2496 \cdot 10^{4}$
$5.0 \cdot 10^{-9}$	$1.00 \cdot 10^{4}$	$1.0670 \cdot 10^{4}$
$6.0 \cdot 10^{-9}$	$8.80 \cdot 10^{3}$	$9.3132 \cdot 10^{3}$
$7.0 \cdot 10^{-9}$	$8.00 \cdot 10^{3}$	$8.2648 \cdot 10^{3}$
$8.0 \cdot 10^{-9}$	$7.40 \cdot 10^{3}$	$7.4296 \cdot 10^{3}$
$9.0 \cdot 10^{-9}$	$7.00 \cdot 10^{3}$	$6.7483 \cdot 10^{3}$

[1]*W. T. Wyatt, Internal EMP Strength and Time Dependence for an Idealized Problem, Report 1994, U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, VA (February 1971).*

made with different time step sizes, which revealed some interesting facts. When the time step size $\Delta t = 1.0 \cdot 10^{-10}$ was used, stability of the solution was obtained from 0 to approximately 6.5 ns, while from 6 to 9 ns, the solution became dominated by very large truncation errors and, thus, resulted in an unstable solution over this time frame. Using the time step sizes $\Delta t = 1.0 \cdot 10^{-11}$ and $\Delta t = 1.0 \cdot 10^{-12}$ did not produce any stability problems during the time frame 0 to 9 ns, and the truncation errors were considerably smaller than for $\Delta t = 1.0 \cdot 10^{-10}$. The truncation errors did not decrease, however, when the time step size was lowered to $\Delta t = 1.0 \cdot 10^{-13}$, as they remained approximately on the same order of magnitude. An example of these results appears in table II.

TABLE II. HAMMING'S TRUNCATION ERRORS FOR DIFFERENT TIME STEP SIZES

Time (s)	Truncation errors for time step sizes (Δt)			
	$\Delta t = 1.0 \cdot 10^{-10}$	$\Delta t = 1.0 \cdot 10^{-11}$	$\Delta t = 1.0 \cdot 10^{-12}$	$\Delta t = 1.0 \cdot 10^{-13}$
0	0	0	0	0
$1.0 \cdot 10^{-10}$	0	$3.4909 \cdot 10^{-6}$	$-8.6590 \cdot 10^{-11}$	$-1.2123 \cdot 10^{-10}$
$2.0 \cdot 10^{-10}$	0	$4.1106 \cdot 10^{-6}$	$-1.7318 \cdot 10^{-11}$	$-6.9272 \cdot 10^{-11}$
$3.0 \cdot 10^{-10}$	0	$4.0913 \cdot 10^{-6}$	$-5.1954 \cdot 10^{-11}$	$-1.2123 \cdot 10^{-10}$
$4.0 \cdot 10^{-10}$	$1.9735 \cdot 10^{-1}$	$3.6824 \cdot 10^{-6}$	$-6.9272 \cdot 10^{-11}$	$-1.3854 \cdot 10^{-10}$
$1.0 \cdot 10^{-9}$	$8.5146 \cdot 10^{-2}$	$5.5282 \cdot 10^{-7}$	$-6.9272 \cdot 10^{-11}$	$-6.0613 \cdot 10^{-11}$
$2.0 \cdot 10^{-9}$	$-1.1649 \cdot 10^{-3}$	$-1.5846 \cdot 10^{-8}$	$-5.1954 \cdot 10^{-11}$	$-4.3295\ 10^{-11}$
$3.0 \cdot 10^{-9}$	$-1.7558 \cdot 10^{-4}$	$-3.2038 \cdot 10^{-9}$	$-6.4942 \cdot 10^{-11}$	$-5.1954 \cdot 10^{-11}$
$4.0 \cdot 10^{-9}$	$-1.0871 \cdot 10^{-4}$	$-8.9188 \cdot 10^{-10}$	$-3.8965 \cdot 10^{-11}$	$-2.5977 \cdot 10^{-11}$
$5.0 \cdot 10^{-9}$	$-8.4018 \cdot 10^{-3}$	$-3.4203 \cdot 10^{-10}$	$-2.1647 \cdot 10^{-11}$	$-3.0306 \cdot 10^{-11}$
$6.0 \cdot 10^{-9}$	$-1.2692 \cdot 10^{1}$	$-1.6019 \cdot 10^{-10}$	$-3.0306 \cdot 10^{-11}$	$-3.0306 \cdot 10^{-11}$
$7.0 \cdot 10^{-9}$	$-2.3989 \cdot 10^{5}$	$-8.6590 \cdot 10^{-11}$	$-3.4636 \cdot 10^{-11}$	$-3.0306 \cdot 10^{-11}$
$8.0 \cdot 10^{-9}$	$-5.0072 \cdot 10^{10}$	$-6.4942 \cdot 10^{-11}$	$-1.9483 \cdot 10^{-11}$	$-2.1647 \cdot 10^{-11}$
$9.0 \cdot 10^{-9}$	$-9.8303 \cdot 10^{16}$	$-3.2471 \cdot 10^{-11}$	$-1.2988 \cdot 10^{-11}$	$-1.0824 \cdot 10^{-11}$

Some interesting occurrences were noted when analyzing the results of EXPDIFF. First, EXPDIFF was very simple to code. In fact, because of the relative ease and simplicity in programming EXPDIFF, valid results were obtained on the first computer run. Second, the results of EXPDIFF were quite favorable when compared to the results obtained by Wyatt[1] and again are considered more accurate because of the calculation

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[1] *W. T. Wyatt, Internal EMP Strength and Time Dependence for an Idealized Problem, Report 1994, U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, VA (February 1971).*

TABLE III. RESULTS OF ORIGINAL, HAMMING, AND EXPDIFF

Time (s)	Original Electric field strength (V/m)	HAMMING Electric field strength $\Delta t = 1.0 \cdot 10^{-12}$ (V/m)	EXPDIFF Electric field strength $\Delta t = 1.0 \quad 10^{-12}$ (V/m)
$-4.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6970 \cdot 10^{4}$	$3.6970 \cdot 10^{4}$
$-3.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6965 \cdot 10^{4}$	$3.6965 \cdot 10^{4}$
$-2.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6961 \cdot 10^{4}$	$3.6961 \cdot 10^{4}$
$-1.0 \cdot 10^{-9}$	$3.80 \cdot 10^{4}$	$3.6960 \cdot 10^{4}$	$3.6960 \cdot 10^{4}$
0	$3.80 \cdot 10^{4}$	$3.6959 \cdot 10^{4}$	$3.6959 \cdot 10^{4}$
$5.0 \cdot 10^{-10}$	$3.00 \cdot 10^{4}$	$3.2129 \cdot 10^{4}$	$3.2135 \cdot 10^{4}$
$1.0 \cdot 10^{-9}$	$2.48 \cdot 10^{4}$	$2.6259 \cdot 10^{4}$	$2.6264 \cdot 10^{4}$
$1.5 \cdot 10^{-9}$	$2.10 \cdot 10^{4}$	$2.2092 \cdot 10^{4}$	$2.2095 \cdot 10^{4}$
$2.0 \cdot 10^{-9}$	$1.85 \cdot 10^{4}$	$1.9107 \cdot 10^{4}$	$1.9110 \cdot 10^{4}$
$3.0 \cdot 10^{-9}$	$1.47 \cdot 10^{4}$	$1.5093 \cdot 10^{4}$	$1.5094 \cdot 10^{4}$
$4.0 \cdot 10^{-9}$	$1.20 \cdot 10^{4}$	$1.2496 \cdot 10^{4}$	$1.2497 \cdot 10^{4}$
$5.0 \cdot 10^{-9}$	$1.00 \cdot 10^{4}$	$1.0670 \cdot 10^{4}$	$1.0670 \cdot 10^{4}$
$6.0 \cdot 10^{-9}$	$8.80 \cdot 10^{3}$	$9.3132 \cdot 10^{3}$	$9.3138 \cdot 10^{3}$
$7.0 \cdot 10^{-9}$	$8.00 \cdot 10^{3}$	$8.2648 \cdot 10^{3}$	$8.2652 \cdot 10^{3}$
$8.0 \cdot 10^{-9}$	$7.40 \cdot 10^{3}$	$7.4296 \cdot 10^{3}$	$7.4299 \cdot 10^{3}$
$9.0 \cdot 10^{-9}$	$7.00 \cdot 10^{3}$	$6.7483 \cdot 10^{3}$	$6.7486 \cdot 10^{3}$

and analysis of the truncation errors (table III). Probably the most significant aspect of EXPDIFF was the results of comparisons with HAMMING.

In comparing the codes, several significant points of interest were revealed. There was a noticeable ease in the programming of EXPDIFF compared with the more difficult effort required for HAMMING. In fact, a very generous analysis of this coding effort was that HAMMING took at least twice as long to code as EXPDIFF. Although HAMMING is generally considered to give more accurate calculations, EXPDIFF yielded results that were extremely close (sometimes exact) to the solutions obtained from HAMMING (table III). However, the most significant result was the stability of EXPDIFF when compared with HAMMING. For the time step size $\Delta t = 1.0 \cdot 10^{-11}$, the truncation errors were approximately the same size, except in several cases where truncation errors from HAMMING were

an order of magnitude smaller. When the time step size was increased to $\Delta t = 1.0 \cdot 10^{-10}$, HAMMING became unstable at about 6.5 ns, while EXPDIFF yielded truncation errors on the order of 10^{-6} for 0 to 9 ns. This timing is illustrated in table IV. An even more meaningful result was noticed when considering the large step size of $\Delta t = 1.0 \cdot 10^{-9}$. For this step size, EXPDIFF was stable through 9 ns, since truncation errors on the order of 10^{-2} were tolerated. Whereas EXPDIFF exhibited reasonable truncation errors for this step size, HAMMING, as expected, became dominated by the buildup of truncation errors and, thus, was unstable. The truncation errors of EXPDIFF for $\Delta t = 1.0 \cdot 10^{-9}$ can be seen in table V.

TABLE IV. TRUNCATION ERRORS FOR DIFFERENT TIME STEP SIZES FOR HAMMING AND EXPDIFF

Time (s)	Truncation errors at time step size $1.0 \cdot 10^{-10}$		Truncation errors at time step size $1.0 \cdot 10^{-11}$	
	HAMMING	EXPDIFF	HAMMING	EXPDIFF
0	0	$7.4679 \cdot 10^{-5}$	0	$5.0168 \cdot 10^{-8}$
$1.0 \cdot 10^{-10}$	0	$5.9718 \cdot 10^{-11}$	$3.4909 \cdot 10^{-6}$	$3.9421 \cdot 10^{-9}$
$2.0 \cdot 10^{-10}$	0	$5.1673 \cdot 10^{-6}$	$4.1106 \cdot 10^{-6}$	$6.740 \cdot 10^{-9}$
$3.0 \cdot 10^{-10}$	0	$8.4422 \cdot 10^{-6}$	$4.0913 \cdot 10^{-6}$	$8.3.3 \cdot 10^{-9}$
$4.0 \cdot 10^{-10}$	$1.9735 \cdot 10^{-1}$	$1.0322 \cdot 10^{-5}$	$3.6824 \cdot 10^{-6}$	$9.0334 \cdot 10^{-9}$
$1.0 \cdot 10^{-9}$	$8.5146 \cdot 10^{-2}$	$9.7018 \cdot 10^{-6}$	$5.5282 \cdot 10^{-7}$	$7.0506 \cdot 10^{-9}$
$2.0 \cdot 10^{-9}$	$-1.1649 \cdot 10^{-3}$	$5.3903 \cdot 10^{-6}$	$-1.5846 \cdot 10^{-8}$	$3.6655 \cdot 10^{-9}$
$3.0 \cdot 10^{-9}$	$-1.7558 \cdot 10^{-4}$	$3.5640 \cdot 10^{-6}$	$-3.2038 \cdot 10^{-9}$	$2.2718 \cdot 10^{-9}$
$4.0 \cdot 10^{-9}$	$-1.0871 \cdot 10^{-4}$	$2.6160 \cdot 10^{-6}$	$-8.9188 \cdot 10^{-10}$	$1.5602 \cdot 10^{-9}$
$5.0 \cdot 10^{-9}$	$-8.4018 \cdot 10^{-3}$	$2.0477 \cdot 10^{-6}$	$-3.4203 \cdot 10^{-10}$	$1.1436 \cdot 10^{-9}$
$6.0 \cdot 10^{-9}$	$-1.2692 \cdot 10^{1}$	$1.6749 \cdot 10^{-6}$	$-1.6019 \cdot 10^{-10}$	$8.7743 \cdot 10^{-10}$
$7.0 \cdot 10^{-9}$	$-2.3989 \cdot 10^{5}$	$1.4142 \cdot 10^{-6}$	$-8.6590 \cdot 10^{-11}$	$6.9648 \cdot 10^{-10}$
$8.0 \cdot 10^{-9}$	$-5.0072 \cdot 10^{10}$	$1.2230 \cdot 10^{-6}$	$-6.4942 \cdot 10^{-11}$	$5.6760 \cdot 10^{-10}$
$9.0 \cdot 10^{-9}$	$-9.8303 \cdot 10^{16}$	$1.0775 \cdot 10^{-6}$	$-3.2471 \cdot 10^{-11}$	$4.7242 \cdot 10^{-10}$

TABLE V. EXPDIFF TRUNCATION ERRORS AT TIME STEP SIZE $1.0 \cdot 10^{-9}$

Time (s)	Truncation errors
0	$8.7131 \cdot 10^{-1}$
$1.0 \cdot 10^{-9}$	$2.1944 \cdot 10^{-7}$
$2.0 \cdot 10^{-9}$	$5.1673 \cdot 10^{-2}$
$3.0 \cdot 10^{-9}$	$3.4598 \cdot 10^{-2}$
$4.0 \cdot 10^{-9}$	$2.5851 \cdot 10^{-2}$
$5.0 \cdot 10^{-9}$	$2.0672 \cdot 10^{-2}$
$6.0 \cdot 10^{-9}$	$1.7226 \cdot 10^{-2}$
$7.0 \cdot 10^{-9}$	$1.4766 \cdot 10^{-2}$
$8.0 \cdot 10^{-9}$	$1.2920 \cdot 10^{-2}$
$9.0 \cdot 10^{-9}$	$1.1484 \cdot 10^{-2}$

8. CONCLUSIONS

With regard to the comparisons made between the predictor-corrector routine HAMMING and the exponential differencing method EXPDIFF, there seem to be two compelling factors that make the exponential differencing technique superior. First, the ease and simplicity of the programming effort required for EXPDIFF far outweigh the more complicated coding work needed for HAMMING. Second and probably more important, the stability properties of EXPDIFF are excellent, whereas the solutions calculated by HAMMING became dominated by truncation errors during the time frame examined for particular time step sizes. This stability property is best exemplified by the large step size ($1.0 \cdot 10^{-9}$) that can be used with relative assuredness of accurate results. Thus, this stability factor represented by the reasonable truncation errors of EXPDIFF far outweigh the somewhat smaller truncation errors of HAMMING. Therefore, considering the ease of programming and the stability properties, the exponential differencing method provides an efficient and reliable solution to the differential equations involved in this EMP problem and is strongly recommended for solving other differential equations of this type.

SYMBOLS

$E(t)$	Electric-field strength
ϵ	Permittivity of free space
$J_c(t)$	Compton electron current density
μ	Electron mobility
$N_e(t)$	Free electron density
q	Electronic charge
r	Model parameter for gamma flux rate history
s	Rate of production of ion pairs per unit volume
t	Time

APPENDIX A.--COMPUTER CODE HAMMING

This appendix contains a complete listing of HAMMING.

```
PROGRAM HAMMING    74/74   OPT=1                    FTN 4.5+414       06/04/76  13.30.02       PAGE     1

      PROGRAM HAMMING(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      INTEGER FLAG
      REAL NORD(10),NABS(10)
      COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
      COMMON/B/VAL(5000),CF(5000)
      COMMON/C/RNE(5000),SNE(5000)
      DIMENSION K(5000),TITLE(20)
      DIMENSION ERROR(5000)
      DIMENSION TERR(5000)
      DIMENSION ECOREC(60),ECORECP(60)
   10 L=1
      IT=0
      READ(5,20)DEL,TMAX,EPSLON,START,ESTART,M
   20 FORMAT(5E10.3,I5)
      CF(1)=EPSLON
C
C  CALCULATE STARTING VALUES Y1,Y2,Y3,USING RUNGE-KUTTA TECHNIQUE
C
      CALL RUNGE
      DO 30 J=1,4
      CF(J)=EPSLON
   30 EP(J)=FET(H(J),E(J),J)
      I=4
C
C  PREDICT Y(N+1) USING MILNE"S PREDICTOR
C
   40 CALL PREDICT(I,EPDIC)
      CF(I+1)=EPSLON
      H(I+1)=H(I)+DEL
      EPDICP=FET(H(I+1),EPDIC,0)
      ECOREC(L)=EPDIC
      ECORECP(L)=EPDICP
C
C  CORRECT Y(N+1) USING HAMMING"S CORRECTOR
C
   50 CALL CORRECT(I,ECORECP(L),ECOREC(L+1))
      ECORECP(L+1)=FET(H(I+1),ECOREC(L),0)
      IF(L.EQ.1)  GO TO 55
C
C  CHECK TO SEE IF ERROR RESULTS FROM NOT ITERATING TO CONVERGENCE
C
      CALL EROR(I,L,EPDIC,ECOREC(L),ECOREC(L+1),ECORECP(L),
     1ECORECP(L+1),FLAG,IT)
      IF(FLAG) 53,60,60
   53 ERROR(I+1)=ABS(ECORECP(L+1)-ECORECP(L))
      IF(L.GT.50)  GO TO 65
   55 L=L+1
      GO TO 50
   60 K(I+1)=L-1
C
C  CALCULATE CORRECT VALUE OF Y(N+1) USING HAMMING"S CORRECTOR
C
      CALL CORRECT(I,ECORECP(L+1),ECOR)
      E(I+1)=ECOR
      EP(I+1)=FET(H(I+1),E(I+1),I+1)
C
C  CALCULATE TRUNCATION ERROR
```

```
PROGRAM HAMMING     74/74   OPT=1                        FTN 4.5+414        06/04/76  13.30.02        PAGE     2

      C
            TERR(I+1)=(9./121.)*(E(I+1)-EPDIC)
            N=I+1
            I=I+1
            L=1
            IF(H(I).LE.TMAX)  GO TO 40
         65 WRITE(6,70)
         70 FORMAT(1H1)
            WRITE(6,80) DEL,TMAX,M,EPSLON,START
         80 FORMAT(1X,17HDELTA TIME STEP: ,1PE10.3,5X,
           120HMAXIMUM TIME TO BE PLOTTED: ,1PE10.3,5X,
           220HINCREMENT OF OUTPUT VALUES: ,I5,/,
           344X,23HCONVERGENCE TOLERANCE: ,1PE10.3,
           45X,15HSTARTING TIME: ,1PE10.3,/,/,/,/)
            IF(L.GT.50.AND.FLAG.EQ.-1)  GO TO 140
            WRITE(6,90)
         90 FORMAT(/,/,/,53X,9HNUMBER OF,10X,8HERROR BY)
            WRITE(6,100)
        100 FORMAT(4X,4HTIME,14X,1HE,7X,16HTRUNCATION ERROR,3X,
           118HCONVERGENCE CYCLES,3X,13HNOT ITERATING,3X,
           218HCONVERGENCE FACTOR,5X,6HOLD NE,8X,6HNEW NE)
            WRITE(6,110)
        110 FORMAT(4X,4H----,14X,1H-,7X,16H----------------,3X,
           118H------------------,3X,13H-------------,3X,
           218H------------------,5X,6H------,8X,6H------)
            DO 130 J=1,N,M
            WRITE(6,120) H(J),E(J),TERR(J),K(J),VAL(J),CF(J),RNE(J),SNE(J)
        120 FORMAT(1X,3(1PE11.4,5X),7X,I3,11X,1PE11.4,8X,1PE11.4,7X,1PE11.4,
           13X,1PE11.4)
        130 CONTINUE
      C
      C
            TITLE(1)=10HEMP TIME D
            TITLE(2)=10HEPENDENCY
            TITLE(3)=5HSTUDY
            TITLE(4)=1H
            TITLE(5)=1H
            NORD(1)=10HELECTRIC F
            NORD(2)=10HIELD STREN
            NORD(3)=10HGTH  V/M
            NABS(1)=4HTIME
            NABS(2)=1H
            NABS(3)=1H
      C
      C
            CALL PLOT(N,MAX)
            CALL PRTPLT(H,E,MAX,TITLE,NABS,NORD,0.,0.,100)
            GO TO 155
        140 KL=I+1
            WRITE(6,150) KL,ERROR(KL)
        150 FORMAT(/,/,/,/,1X44HDID NOT CONVERGE IN 50 ITERATIONS WHILE CALC,
           110HULATING E(,I5,25H)  -  CONVERGENCE ERROR: ,1PE11.4)
        155 READ(5,160) IK
        160 FORMAT(9X,I1)
            GO TO (10,170),IK
        170 CONTINUE
            END
```

```
          PROGRAM HAMMING    74/74   OPT=1                     FTN 4.5+414        06/04/76  13.30.02        PAGE    3
CARD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM
   112     I                AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.


      SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE     REFERENCES
 4111  HAMMING        1

VARIABLES      SN  TYPE             RELOCATION
11610  CF          REAL      ARRAY    B          REFS         5        84   DEFINED       15        21        28
35230  DEL         REAL               A          REFS         4        29        66  DEFINED        13
    0  E           REAL      ARRAY    A          REFS         4        22        55       59        84       104
                                              DEFINED        54
 4645  ECOR        REAL                          REFS        53        54
42152  ECOREC      REAL      ARRAY               REFS        10        36        37      2*42  DEFINED        31
42246  ECORECP     REAL      ARRAY               REFS        10        36      2*42      2*45       53
                                              DEFINED        32        37
23420  EP          REAL      ARRAY    A          REFS         4   DEFINED        22       55
 4643  EPDIC       REAL                          REFS        27        30        31       42        59
 4644  EPDICP      REAL                          REFS        32   DEFINED        30
 4637  EPSLON      REAL                          REFS        15        21        28       66   DEFINED        13
16532  ERROR       REAL      ARRAY               REFS         8       107   DEFINED       45
35232  ESTART      REAL               A          REFS         4   DEFINED        13
 4633  FLAG        INTEGER                       REFS         2        42        44       72
11610  H           REAL      ARRAY    A          REFS         4        22        29       30        37        55        63
                                                   84       104   DEFINED        29
 4642  I           INTEGER                       REFS        27        28      2*29       30        36        37        42
                                                   45        49        53        54     4*55      2*59        60        61
                                                   63       106   DEFINED        23       61
 4651  IK          INTEGER                       REFS       112   DEFINED       110
 4635  IT          INTEGER                       REFS        42   DEFINED        12
 4641  J           INTEGER                       REFS        21      4*22      8*84  DEFINED        20        83
 4676  K           INTEGER   ARRAY               REFS         7        84   DEFINED       49
 4650  KL          INTEGER                       REFS     2*107   DEFINED       106
 4634  L           INTEGER                       REFS        31        32      2*36     2*37        38      5*42      2*45
                                                   46        47        49        53       72   DEFINED        11        47
                                                   62
 4640  M           INTEGER                       REFS        66        83   DEFINED       13
 4647  MAX         INTEGER                       REFS       103       104
 4646  N           INTEGER                       REFS        83       103   DEFINED       60
 4664  NABS        REAL      ARRAY               REFS         3       104   DEFINED       98        99       100
 4652  NORD        REAL      ARRAY               REFS         3       104   DEFINED       95        96        97
    0  RNE         REAL      ARRAY    C          REFS         6        84
11610  SNE         REAL      ARRAY    C          REFS         6        84
35231  START       REAL               A          REFS         4        66   DEFINED       13
30342  TERR        REAL      ARRAY               REFS         9        84   DEFINED       59
16506  TITLE       REAL      ARRAY               REFS         7       104   DEFINED       90        91        92        93
                                                   94
 4636  TMAX        REAL                          REFS        63        66   DEFINED       13
    0  VAL         REAL      ARRAY    B          REFS         5        84
```

```
        PROGRAM HAMMING    74/74   OPT=1                        FTN 4.5+414        06/04/76  13.30.02       PAGE     4

FILE NAMES       MODE
     0  INPUT
  2041  OUTPUT
     0  TAPE5    FMT                  READS      13        110
  2041  TAPE6    FMT                  WRITES     64         66         73         75         79         84        107

EXTERNALS          TYPE    ARGS       REFERENCES
        CORRECT               3              36     53
        EROR                  9              42
        FET        REAL       3              22     30         37         55
        PLOT                  2             103
        PREDICT               2              27
        PRTPLT                9             104
        RUNGE                 0              19

INLINE FUNCTIONS   TYPE    ARGS          DEF LINE  REFERENCES
        ABS        REAL       1  INTRIN                 45

STATEMENT LABELS             DEF LINE   REFERENCES
  4112  10                       11        112
  4425  20       FMT             14         13
     0  30                       22         20
  4134  40                       27         63
  4150  50                       36         48
     0  53        INACTIVE       45         44
  4200  55                       47         38
  4202  60                       49       2*44
  4226  65                       64         46
  4433  70       FMT             65         64
  4445  80       FMT             67         66
  4473  90       FMT             74         73
  4503  100      FMT             76         75
  4524  110      FMT             80         79
  4555  120      FMT             85         84
     0  130                      87         83
  4310  140                     106         72
  4572  150      FMT            108        107
  4316  155                     110        105
  4613  160      FMT            111        110
  4327  170                     113        112

LOOPS  LABEL    INDEX     FROM-TO    LENGTH    PROPERTIES
  4122  30      * J        20 22       11B              EXT REFS
  4244  130     * J        83 87       23B              EXT REFS

COMMON BLOCKS   LENGTH     MEMBERS - BIAS NAME(LENGTH)
        A        15003              0 E      (5000)          5000 H      (5000)          10000 EP     (5000)
                                15000 DEL    (1)            15001 START  (1)             15002 ESTART (1)
        B        10000              0 VAL    (5000)          5000 CF     (5000)
        C        10000              0 PNE    (5000)          5000 SNE    (5000)

STATISTICS
  PROGRAM LENGTH                362378    15519
  BUFFER LENGTH                  41038     2115
  CM LABELED COMMON LENGTH     1042738    35003
```

APPENDIX A

```
SUBROUTINE RUNGE      74/74    OPT=1                         FTN 4.5+414        06/04/76  13.30.02        PAGE      1

    1               SUBROUTINE RUNGE
                    COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
                    H(1)=START
                    E(1)=ESTART
    5               I=1
                 10 R1=FET(H(I),E(I),0)*DEL
                    T2=H(I)+.5*DEL
                    E2=E(I)+.5*R1
                    R2=FET(T2,E2,0)*DEL
   10               E3=E(I)+.5*R2
                    R3=FET(T2,E3,0)*DEL
                    T4=H(I)+DEL
                    E4=E(I)+R3
                    R4=FET(T4,E4,0)*DEL
   15               DELE=(R1+2.*R2+2.*R3+R4)/6.
                    E(I+1)=E(I)+DELE
                    I=I+1
                    H(I)=H(I-1)+DEL
                    IF(I.GT.3)  RETURN
   20               GO TO 10
                    END


    SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS     DEF LINE     REFERENCES
    1 RUNGE          1           19

VARIABLES     SN  TYPE            RELOCATION
35230 DEL         REAL                 A          REFS      2       6       7       9      11      12      14
                                                    18
  113 DELE        REAL                            REFS     16   DEFINED    15
    0 E           REAL       ARRAY     A          REFS      2       6       8      10      13      16
                                                DEFINED     4      16
23420 EP          REAL       ARRAY     A          REFS      2
35232 ESTART      REAL                 A          REFS      2       4
  104 E2          REAL                            REFS      9   DEFINED     8
  106 E3          REAL                            REFS     11   DEFINED    10
  111 E4          REAL                            REFS     14   DEFINED    13
11610 H           REAL       ARRAY     A          REFS      2       6       7      12      18
                                                DEFINED     3      18
  101 I           INTEGER                         REFS    2*6       7       8      10      12      13     2*16
                                                    17   2*18      19  DEFINED      5      17
  102 R1          REAL                            REFS      8      15  DEFINED      6
  105 R2          REAL                            REFS     10      15  DEFINED      9
  107 R3          REAL                            REFS     13      15  DEFINED     11
  112 R4          REAL                            REFS     15   DEFINED    14
35231 START       REAL                 A          REFS      2       3
  103 T2          REAL                            REFS      9      11  DEFINED      7
  110 T4          REAL                            REFS     14   DEFINED    12

EXTERNALS         TYPE    ARGS        REFERENCES
      FET         REAL      3              6       9      11      14
```

```
      SUBROUTINE RUNGE        74/74   OPT=1                       FTN 4.5+414        06/04/76  13.30.02         PAGE     2

STATEMENT LABELS           DEF LINE    REFERENCES
    6  10                       6          20

COMMON BLOCKS   LENGTH      MEMBERS - BIAS NAME(LENGTH)
    A           15003                 0 E      (5000)         5000 H      (5000)          10000 EP      (5000)
                                  15000 DEL    (1)           15001 START  (1)             15002 ESTART  (1)

STATISTICS
  PROGRAM LENGTH                  114B        76
  CM LABELED COMMON LENGTH      35233B     15003
```

```
      SUBROUTINE PREDICT    74/74   OPT=1                        FTN 4.5+414       06/04/76  13.30.02       PAGE     1

 1                    SUBROUTINE PREDICT(I,EPDIC)
                      COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
                      V=2.*EP(I)-EP(I-1)+2.*EP(I-2)
                      EPDIC=E(I-3)+((4.*DEL)/3.)*V
 5                    RETURN
                      END


      SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS     DEF LINE      REFERENCES
    3 PREDICT         1             5

VARIABLES      SN  TYPE              RELOCATION
35230 DEL          REAL                A              REFS          2          4
    0 E            REAL        ARRAY   A              REFS          2          4
29420 EP           REAL        ARRAY   A              REFS          2         3*3
    0 EPDIC        REAL                  F.P.      DEFINED          1          4
35232 ESTART       REAL                A              REFS          2
11610 H            REAL        ARRAY   A              REFS          2
    0 I            INTEGER               F.P.         REFS         3*3         4    DEFINED        1
35231 START        REAL                A              REFS          2
   22 V            REAL                               REFS          4    DEFINED        3

COMMON BLOCKS    LENGTH      MEMBERS - BIAS NAME(LENGTH)
    A            15003                 0 E        (5000)            5000 H      (5000)          10000 EP      (5000)
                                   15000 DEL      (1)              15001 START  (1)             15002 ESTART  (1)

STATISTICS
   PROGRAM LENGTH                        23B       19
  CM LABELED COMMON LENGTH           35233B     15003
```

APPENDIX A

```
SUBROUTINE CORRECT     74/74    OPT=1                                FTN 4.5+414         06/04/76  13.30.02          PAGE      1

 1                   SUBROUTINE CORRECT(I,EDICP,ECREC)
                     COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
                     V1=(9.*E(I)-E(I-2))/8.
                     V2=EDICP+2.*EP(I)-EP(I-1)
 5                   ECREC=V1+((3.*DEL)/8.)*V2
                     RETURN
                     END


     SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE      REFERENCES
    3 CORRECT          1             6

VARIABLES       SN  TYPE                RELOCATION
35230  DEL          REAL                    A               REFS        2          5
    0  E            REAL          ARRAY     A               REFS        2          2*3
    0  ECREC        REAL                       F.P.     DEFINED         1          5
    0  EDICP        REAL                       F.P.         REFS        4     DEFINED          1
23420  EP           REAL          ARRAY     A               REFS        2          2*4
35232  ESTART       REAL                    A               REFS        2
11610  H            REAL          ARRAY     A               REFS        2
    0  I            INTEGER                    F.P.         REFS        2*3        2*4    DEFINED          1
35231  START        REAL                    A               REFS        2
   26  V1           REAL                                    REFS        5     DEFINED          3
   27  V2           REAL                                    REFS        5     DEFINED          4

COMMON BLOCKS    LENGTH      MEMBERS - BIAS NAME(LENGTH)
     A           15003               0 E        (5000)               5000 H       (5000)               10000 EP       (5000)
                                 15000 DEL      (1)                 15001 START   (1)                 15002 ESTART   (1)

STATISTICS
  PROGRAM LENGTH                 30B       24
  CM LABELED COMMON LENGTH    35233B    15003
```

```
SUBROUTINE ERDR       74/74   OPT=1                    FTN 4.5+414      06/04/76  13.30.02      PAGE     1

          SUBROUTINE ERDR(I,L,EPDIC,P,C,PP,CP,FLAG,IT)
          COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
          COMMON/B/VAL(5000),CF(5000)
          INTEGER FLAG
          B=3./8.
          DELTA=ABS(CP-PP)
     10   IF(DELTA.LE.CF(I+1))  GO TO 20
          IF(L.LE.10)  GO TO 15
          CF(I+1)=DELTA*CF(I+1)
     15   FLAG=-1
          RETURN
     20   ARG=C-P
          IF(ARG.EQ.0.)  ARG=1.0E-5
          RK=DELTA/ABS(ARG)
          VAL(I+1)=(DEL**2)*(B**2)*DELTA*RK
          CALL CORRECT(I,PP,C)
          EC=C
          ERR=(9./121.)*(EC-EPDIC)
          IF(VAL(I+1).GT.ABS(ERR))  GO TO 40
          FLAG=0
          RETURN
     40   IP1=I+1
          IF(IT.GT.0)  GO TO 80
          WRITE(6,60)
     60   FORMAT(1H1)
          WRITE(6,70)
     70   FORMAT(/,/,/,1X,40HTHE ERROR INCURRED BY NOT ITERATING THE ,
         161HCORRECTOR TO CONVERGENCE IS GREATER THAN THE TRUNCATION ERROR,
         2/,16X,12HERROR BY NOT,/,16X,12HITERATING TO,5X,10HTRUNCATION,
         3/,6X,4HE(I),6X,11HCONVERGENCE,6X,5HERROR)
          IT=1
     80   WRITE(6,90) IP1,VAL(I+1),ERR
     90   FORMAT(5X,2HE(,I4,1H),5X,1PE11.4,5X,1PE11.4)
          FLAG=1
          RETURN
          END
```

```
SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS     DEF LINE     REFERENCES
   3 ERDR            1           11        21         35

VARIABLES      SN  TYPE          RELOCATION
   162 ARG         REAL                          REFS     13        14  DEFINED       12        13
   160 B           REAL                          REFS     15   DEFINED         5
     0 C           REAL                   F.P.   REFS     12        16        17  DEFINED        1
 11610 CF          REAL     ARRAY      B         REFS      3         7         9  DEFINED        9
     0 CP          REAL                   F.P.   REFS      6   DEFINED         1
 35230 DEL         REAL                A         REFS      2        15
   161 DELTA       REAL                          REFS      7         9        14        15  DEFINED        6
     0 E           REAL     ARRAY      A         REFS      2
   164 EC          REAL                          REFS     18   DEFINED        17
 23420 EP          REAL     ARRAY      A         REFS      2
```

APPENDIX A

```
      SUBROUTINE EROR       74/74    OPT=1                               FTN 4.5+414       06/04/76  13.30.02        PAGE     2

VARIABLES      SN  TYPE            RELOCATION
     0 EPDIC      REAL                  F.P.      REFS       10  DEFINED        1
   165 ERR        REAL                            REFS       19       32  DEFINED       18
 35232 ESTART     REAL              A             REFS        2
     0 FLAG       INTEGER               F.P.      REFS        4  DEFINED        1       10       20       34
 11610 H          REAL       ARRAY  A             REFS        2
     0 I          INTEGER               F.P.      REFS        7      2*9       15       16       19       22       32
                                               DEFINED        1
   166 IP1        INTEGER                         REFS       32  DEFINED       22
     0 IT         INTEGER               F.P.      REFS       23  DEFINED        1       31
     0 L          INTEGER               F.P.      REFS        8  DEFINED        1
     0 P          REAL                  F.P.      REFS       12  DEFINED        1
     0 PP         REAL                  F.P.      REFS        6       16  DEFINED        1
   163 RK         REAL                            REFS       15  DEFINED       14
 35231 START      REAL              A             REFS        2
     0 VAL        REAL       ARRAY  B             REFS        3       19       32  DEFINED       15

FILE NAMES        MODE
      TAPE6       FMT                  WRITES        24       26       32

EXTERNALS              TYPE   ARGS         REFERENCES
      CORRECT                  3             16

INLINE FUNCTIONS   TYPE   ARGS          DEF LINE  REFERENCES
      ABS          REAL      1  INTRIN             6       14       19

STATEMENT LABELS               DEF LINE   REFERENCES
     0 10          INACTIVE          7
    17 15                           10          8
    21 20                           12          7
    52 40                           22         19
   101 60        FMT                25         24
   106 70        FMT                27         26
    63 80                           32         23
   142 90        FMT                33         32

COMMON BLOCKS     LENGTH      MEMBERS - BIAS NAME(LENGTH)
      A            15003                 0 E       (5000)             5000 H       (5000)           10000 EP      (5000)
                                     15000 DEL     (1)               15001 START   (1)              15002 ESTART  (1)
      B            10000                 0 VAL     (5000)             5000 CF      (5000)

STATISTICS
  PROGRAM LENGTH                   167B       119
  CM LABELED COMMON LENGTH       60653B     25003
```

```
SUBROUTINE PLOT       74/74   OPT=1                          FTN 4.5+414        06/04/76  13.30.02       PAGE     1

     1                    SUBROUTINE PLOT(N,MAX)
                          COMMON/A/E(5000),H(5000),EP(5000),DEL,START,ESTART
                          RT=N/100.
                          IDEL=INT(RT)
     5                    IF(IDEL.EQ.0)  IDEL=1
                          L=1
                          DO 10 I=1,N,IDEL
                          H(L)=H(I)
                          E(L)=E(I)
    10                    L=L+1
                       10 CONTINUE
                          MAX=L
                          RETURN
                          END

     SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE      REFERENCES
    3 PLOT            1            13

VARIABLES      SN  TYPE              RELOCATION
35230  DEL         REAL                  A           REFS       2
    0  E           REAL        ARRAY     A           REFS       2        9    DEFINED      9
23420  EP          REAL        ARRAY     A           REFS       2
35232  ESTART      REAL                  A           REFS       2
11610  H           REAL        ARRAY     A           REFS       2        8    DEFINED      8
   34  I           INTEGER                           REFS       8        9    DEFINED      7
   32  IDEL        INTEGER                           REFS       5        7    DEFINED      4       5
   33  L           INTEGER                           REFS       8        9         10     12    DEFINED      6       10
    0  MAX         INTEGER                F.P.     DEFINED      1       12
    0  N           INTEGER                F.P.       REFS       3        7    DEFINED      1
   31  RT          REAL                              REFS       4    DEFINED       3
35231  START       REAL                  A           REFS       2

INLINE FUNCTIONS    TYPE    ARGS        DEF LINE  REFERENCES
      INT          INTEGER    1  INTRIN                  4

STATEMENT LABELS              DEF LINE    REFERENCES
    0 10                         11          7

LOOPS  LABEL     INDEX     FROM-TO    LENGTH    PROPERTIES
  17 10           I          7 11        4B       INSTACK

COMMON BLOCKS   LENGTH      MEMBERS - BIAS NAME(LENGTH)
      A          15003              0 E      (5000)             5000 H      (5000)             10000 EP     (5000)
                                15000 DEL    (1)               15001 START  (1)                15002 ESTART (1)

STATISTICS
  PROGRAM LENGTH                 35B        29
  CM LABELED COMMON LENGTH    35233B     15003
```

APPENDIX A

```
FUNCTION FET        74/74   OPT=1                          FTN 4.5+414      06/04/76  13.30.02        PAGE     1

1                   FUNCTION FET(TI,ET,K)
                    REAL JC,MUP,NE
                    R=5.0E+8
                    EPS=8.85E-12
5                   Q=1.6E-19
                    MUP=2.0
                    IF(TI.GT.0.)  GO TO 10
              C  NE(T) IS MULTIPLIED BY 1.E+6 FOR CORRECTION TO METERS
                    FET=(JC(TI)-1.E+6*NE(TI,K)*Q*MUP*ET*EXP(R*TI))/EPS
10                  RETURN
              C  NE(T) IS MULTIPLIED BY 1.E+6 FOR CORRECTION TO METERS
                 10 FET=(JC(TI)-1.E+6*NE(TI,K)*Q*MUP*ET)/EPS
                    RETURN
                    END


        SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS        DEF LINE     REFERENCES
    4 FET              1           10        13

VARIABLES       SN  TYPE             RELOCATION
   73 EPS           REAL                        REFS          9         12    DEFINED       4
    0 ET            REAL                F.P.    REFS          9         12    DEFINED       1
   70 FET           REAL                     DEFINED          9         12
    0 K             INTEGER             F.P.    REFS          9         12    DEFINED       1
   71 MUP           REAL                        REFS          2          9        12    DEFINED       6
   74 Q             REAL                        REFS          9         12    DEFINED       5
   72 R             REAL                        REFS          9    DEFINED         3
    0 TI            REAL                F.P.    REFS          7        3*9      2*12    DEFINED       1

EXTERNALS           TYPE   ARGS        REFERENCES
      EXP           REAL     1 LIBRARY      9
      JC            REAL     1              2          9        12
      NE            REAL     2              2          9        12

STATEMENT LABELS              DEF LINE    REFERENCES
   36 10                         12            7

STATISTICS
  PROGRAM LENGTH                 75B       61
```

```
FUNCTION NE          74/74    OPT=1                               FTN 4.5+414        06/04/76  13.30.02        PAGE     1

     1              REAL FUNCTION NE(T,K)
                    COMMON/C/RNE(5000),SNE(5000)
                    REAL NEO
                    NEO=1.133E+11
     5              S=5.664E+19
                    IF(T.GE.0.)  GO TO 10
                    NE=1.133E+11
                    RNE(K)=0.
                    SNE(K)=0.
    10              RETURN
                 10 NE=NEO+S*T
                    IF(K.EQ.0)  RETURN
                    SNE(K)=NE
                    RNEO=1.133E+11
    15              RS=1.022E+20
                    DELTA=1.0000E+8
                    A=4.E-7
                    ARG=A*RS*(T**2)
                    X=.5*ARG
    20              Y=DELTA*T
                    Z=-X-Y
                    F1=RS/(2.*DELTA)
                    F2=1.+EXP(X)
                    F3=EXP(Y)-1.
    25              F4=EXP(Z)
                    RNE(K)=(F1*F2*F3+RNEO)*F4
                    RETURN
                    END


   SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS     DEF LINE     REFERENCES
    4  NE           1            10         12        27

VARIABLES     SN  TYPE           RELOCATION
    70  A         REAL                             REFS       18    DEFINED      17
    71  ARG       REAL                             REFS       19    DEFINED      18
    67  DELTA     REAL                             REFS       20         22    DEFINED      16
    75  F1        REAL                             REFS       26    DEFINED      22
    76  F2        REAL                             REFS       26    DEFINED      23
    77  F3        REAL                             REFS       26    DEFINED      24
   100  F4        REAL                             REFS       26    DEFINED      25
     0  K         INTEGER               F.P.       REFS        8          9          12         13        26
                                                DEFINED        1
    62  NE        REAL                             REFS       13    DEFINED       7         11
    63  NEO       REAL                             REFS        3         11    DEFINED       4
     0  RNE       REAL      ARRAY   C              REFS        2    DEFINED       8         26
    65  RNEO      REAL                             REFS       26    DEFINED      14
    66  RS        REAL                             REFS       18         22    DEFINED      15
    64  S         REAL                             REFS       11    DEFINED       5
 11610  SNE       REAL      ARRAY   C              REFS        2    DEFINED       9         13
     0  T         REAL                  F.P.       REFS        6         11         18         20    DEFINED      1
    72  X         REAL                             REFS       21         23    DEFINED      19
```

APPENDIX A

```
      FUNCTION NE         74/74   OPT=1                          FTN 4.5+414       06/04/76  13.30.02        PAGE      2

VARIABLES      SN  TYPE            RELOCATION
    73  V          REAL                            REFS         21         24   DEFINED        20
    74  Z          REAL                            REFS         25    DEFINED         21

EXTERNALS           TYPE    ARGS        REFERENCES
       EXP          REAL      1 LIBRARY      23          24          25

STATEMENT LABELS                DEF LINE    REFERENCES
    15  10                         11           6

COMMON BLOCKS    LENGTH        MEMBERS - BIAS NAME(LENGTH)
       C         10000                   0 RNE     (5000)            5000 SNE    (5000)

STATISTICS
   PROGRAM LENGTH                  101B      65
   CM LABELED COMMON LENGTH     23420B    10000
```

```
FUNCTION JC         74/74   OPT=1                            FTN 4.5+414       06/04/76  13.30.02       PAGE     1

 1                   REAL FUNCTION JC(T)
                     R=5.E+8
                     IF(T.GT.0.)  GO TO 10
                     JC=1.34E+3*EXP(R*T)
 5                   RETURN
                  10 JC=1.34E+3
                     RETURN
                     END


  SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE      REFERENCES
    4  JC             1             5          7

VARIABLES       SN  TYPE             RELOCATION
   20  JC           REAL                          DEFINED        4          6
   21  R            REAL                             REFS        4   DEFINED        2
    0  T            REAL                F.P.         REFS        3          4    DEFINED        1

EXTERNALS           TYPE    ARGS         REFERENCES
      EXP           REAL       1 LIBRARY         4

STATEMENT LABELS              DEF LINE    REFERENCES
   14  10                        6            3

STATISTICS
  PROGRAM LENGTH                 22B         18
```

APPENDIX A

```
SUBROUTINE PRTPLT      74/74    OPT=1                         FTN 4.6+420          10/27/76  09.31.03          PAGE      1

            SUBROUTINE PRTPLT (HORZ,VERT,NPTS,HEAD,HLAB,VLAB,HREF,VREF,J)
      C
      C     THIS FORTRAN SUBROUTINE PRODUCES A FULL PAGE X-Y PRINTER PLOT.
      C
      C     HORZ - REAL ARRAY TO BE PLOTTED ALONG HORIZONTAL AXIS.
      C     VERT - REAL ARRAY TO BE PLOTTED ALONG VERTICAL AXIS.
      C     NPTS - NUMBER OF VALUES IN HORZ,VERT (NTE 101)
      C     HEAD - PLOT HEADING (MUST BE 5 WORDS)
      C     HLAB - LABEL FOR HORIZONTAL AXIS (MUST BE 3 WORDS)
      C     VLAB - LABEL FOR VERTICAL AXIS (MUST BE 3 WORDS)
      C     HREF - REFERENCE VALUE FOR HORIZONTAL AXIS.
      C     VREF - REFERENCE VALUE FOR VERTICAL AXIS.
      C     J  THIS DETERMINES THE SIZE OF THE ARRAYS HORZ AND VERT--NTE 100
      C
            DIMENSION HORZ(J),VERT(J),HEAD(5),HLAB(3),VLAB(3),LINE(101),
           C IVER(101),JHOR(101),HORIZ(101),VERTIC(101),YAXIS(30),DOWNT(21)
            INTEGER BLANK,STAR,DOT,EYE
      C
      C
            DATA BLANK/1H /,STAR/1H*/,DOT/1H./,MINUS/1H-/,EYE/1HI/
            CALL DATE(DAT)
            CALL TIME(TIM)
            HEAD(4)=DAT
            HEAD(5)=TIM
            MO = 1
            NUM = 1
            N = NPTS
            N = MINO(N,101)
            DO 10 I = 1,N
            HORIZ(I) = HORZ(I)
            VERTIC(I) = VERT(I)
         10 CONTINUE
      C
            DECODE (30,1000,VLAB(1)) (YAXIS(I),I=1,30)
       1000 FORMAT(30A1)
         20 NA = N - 1
            DO 30 I = 1,NA
            IF (VERTIC(I).GE.VERTIC(I+1)) GO TO 30
            HOLD = VERTIC(I)
            VERTIC(I) = VERTIC(I+1)
            VERTIC(I+1) = HOLD
            SAVE = HORIZ(I)
            HORIZ(I) = HORIZ(I+1)
            HORIZ(I+1) = SAVE
         30 CONTINUE
            DO 40 I = 1,NA
            IF (VERTIC(I).LT.VERTIC(I+1)) GO TO 20
         40 CONTINUE
            VMAX = VERTIC(1)
            VMIN = VERTIC(N)
            HMAX = HORIZ(1)
            HMIN = HORIZ(1)
            DO 50 I = 2,N
```

```
SUBROUTINE PRTPLT     74/74    OPT=1                    FTN 4.6+420        10/27/76  09.31.03        PAGE     2

          HZ = HORIZ(I)
          HMAX = AMAX1(HMAX,HZ)
          HMIN = AMIN1(HMIN,HZ)
       50 CONTINUE
          DO 60 I = 1,N
          JVER = 50.*(VERTIC(I)-VMIN)/(VMAX-VMIN) + 1.5
          IVER(I) = 52 - JVER
       60 JHOR(I) = 100.*(HORIZ(I)-HMIN)/(HMAX-HMIN) + 1.5
          KHREF = 100.*(HREF-HMIN)/(HMAX-HMIN) + 1.5
          IF (KHREF.LT.1) KHREF = 150
          IF (KHREF.GT.101) KHREF = 150
          KVREF = 50.*(VREF-VMIN)/(VMAX-VMIN) + 1.5
          KVREF = 52 - KVREF
          IF (KVREF.LT.1) KVREF = 1
          IF (KVREF.GT.51) KVREF = 1
          UP = (VMAX-VMIN)/50.
          PRINT 2000 , HEAD
     2000 FORMAT(1H1,35X,5A10//)
          DO 200 K = 1,51
          LINE(1) = EYE
          LINE(101) = EYE
          DO 70 I = 2,100
       70 LINE(I) = BLANK
          IF (K.EQ.1) GO TO 90
          IF (K.EQ.51) GO TO 90
          IF (K.NE.KVREF) GO TO 120
          DO 80 I = 2,100
       80 LINE(I) = DOT
          GO TO 120
       90 DO 100 I = 2,100
      100 LINE(I) = MINUS
          DO 110 I = 6,100,5
      110 LINE(I) = EYE
          GO TO 130
      120 IF (KHREF.EQ.150) GO TO 130
          IF (KHREF.EQ.1) GO TO 130
          LINE(KHREF) = DOT
      130 IF (K.EQ.IVER(NUM)) GO TO 140
          IF (K.LT.IVER(NUM)) GO TO 160
          NUM = NUM + 1
          GO TO 130
      140 IF (JHOR(NUM).GT.101) GO TO 150
          IF (JHOR(NUM).LT.1) GO TO 150
          JH = JHOR(NUM)
          LINE(JH) = STAR
      150 NUMBER = NUM + 1
          IF (IVER(NUM).NE.IVER(NUMBER)) GO TO 160
          NUM = NUMBER
          GO TO 140
      160 T = K - 1
          UPT = VMAX - T*UP
          IF (K.LT.11) GO TO 170
          IF (K.GT.41) GO TO 170
```

```
SUBROUTINE PRTPLT      74/74    OPT=1                        FTN 4.6+420         10/27/76  09.31.03        PAGE     3

                   TAXIS = YAXIS(MQ)
                   MQ = MQ + 1
                   PRINT 3000 , TAXIS,UPT,LINE
              3000 FORMAT(4X,A1,4X,1PE9.2,2X,101A1)
                   GO TO 200
               170 PRINT 4000 , UPT,LINE
              4000 FORMAT(9X,1PE9.2,2X,101A1)
               200 CONTINUE
                   DOWN = (HMAX-HMIN)/20.
                   T = HMIN
                   DO 210 I = 1,21
                   DOWNT(I) = T
                   T = T + DOWN
               210 CONTINUE
                   PRINT 5000 , (DOWNT(I),I=1,21,2)
              5000 FORMAT(/15X,1P11E10.2)
                   PRINT 6000 , (DOWNT(I),I=2,20,2)
              6000 FORMAT(20X,1P10E10.2)
                   PRINT 7000 , HLAB
              7000 FORMAT(/50X,3A10)
                   RETURN
                   END
```

APPENDIX B.--COMPUTER CODE EXPDIFF

This appendix contains a complete listing of EXPDIFF.

```
      PROGRAM EXPDIFF(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL JC,MU,NE,NORD(3),NABS(3)
      COMMON/A/R,EPS,Q,MU,S
      COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
      DIMENSION TITLE(5)
      S=5.664E+19
      R=5.0E+8
      EPS=8.85E-12
      Q=1.6E-19
      MU=2.0
   10 CONTINUE
      I=1
      READ(5,20) DEL,TSTART,TMAX,ESTART,N,J
   20 FORMAT(4E10.3,I5,I5)
      T(1)=TSTART
      E(1)=ESTART
      TERR(1)=0.
      IF(J.NE.1)  GO TO 30
      T(1)=T(N)
      E(1)=E(N)
      TSTART=T(1)
      ESTART=E(N)
   30 CONTINUE
      IF(T(I).GT.TMAX)  GO TO 70
      IF(T(I).GE.0.)  GO TO 50
      CALL FCALC(I,F,FT,FE,FTT,FTE)
      EDP=FT+F*FE
      DELTA(I)=ABS(DEL*FE)
      IF(DELTA(I).LT..1)  GO TO 40
      X=EXP(DEL*FE)-1.-DEL*FE
      E(I+1)=E(I)+DEL*F+EDP*(FE**(-2))*X
      TERR(I+1)=((DEL**3)*(FTT+2.*F*FTE))/6.
      T(I+1)=T(I)+DEL
      I=I+1
      GO TO 30
   40 S1=(DEL**2)/2.
      S2=((DEL**3)*FE)/6.
      S3=((DEL**4)*(FE**2))/24.
      E(I+1)=E(I)+DEL*F+EDP*(S1+S2+S3)
      TERR(I+1)=(DEL**3)*(FTT+2.*F*FTE)/6.
      T(I+1)=T(I)+DEL
      I=I+1
      GO TO 30
   50 CALL GCALC(I,G,GT,GE,GTE)
      EDP=GT+G*GE
      DELTA(I)=ABS(DEL*GE)
      IF(DELTA(I).LT..1)  GO TO 60
      X=EXP(DEL*GE)-1.-DEL*GE
      E(I+1)=E(I)+DEL*G+EDP*(GE**(-2))*X
      TERR(I+1)=(DEL**3)*(2.*G*GTE)/6.
      T(I+1)=T(I)+DEL
      I=I+1
      GO TO 30
   60 S1=(DEL**2)/2.
      S2=((DEL**3)*GE)/6.
      S3=((DEL**4)*(GE**2))/24.
      E(I+1)=E(I)+DEL*G+EDP*(S1+S2+S3)
```

```
PROGRAM EXPDIFF     74/74    OPT=1                          FTN 4.5+414        06/04/76  13.30.36        PAGE      2

                TERR(I+1)=(DEL**3)*(2.*G*GTE)/6.
                T(I+1)=T(I)+DEL
                I=I+1
                GO TO 30
             70 CONTINUE
                N=I-1
                CALL OTWRIT(N,M,DEL,TSTART,TMAX)
        C
        C
                TITLE(1)=10HEMP TIME D
                TITLE(2)=10HEPENDENCY
                TITLE(3)=5HSTUDY
                TITLE(4)=1H
                TITLE(5)=1H
                NORD(1)=10HELECTRIC F
                NORD(2)=10HIELD STREN
                NORD(3)=10HGTH  V/M
                NABS(1)=4HTIME
                NABS(2)=1H
                NABS(3)=1H
        C
        C
                CALL PLOT(N,MAX)
                CALL PRTPLT(T,E,MAX,TITLE,NABS,NORD,0.,0.,100)
                READ(5,80) IK
             80 FORMAT(9X,I1)
                GO TO (10,90),IK
             90 STOP
                END

CARD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM

      84    I              AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.


          SYMBOLIC REFERENCE MAP (R=3)

 ENTRY POINTS     DEF LINE      REFERENCES
  4111  EXPDIFF        1

 VARIABLES     SN  TYPE            RELOCATION
  4455  DEL         REAL                       REFS       28       2*30      31       32       33       36       37
                                                 30       39        40       41       46     2*48       49       50
                                                 51       54        55       56       57       58       59       64
                                            DEFINED       13
 23420  DELTA       REAL       ARRAY    B      REFS        4        29       47   DEFINED      28       46
 11610  E           REAL       ARRAY    B      REFS        4        20       22       31       39       49       57
                                                 81  DEFINED        16       20       31       39       49       57
  4471  EDP         REAL                       REFS       31        39       49       57  DEFINED       27       45
     1  EPS         REAL                A      REFS        3   DEFINED        8
  4460  ESTART      REAL                       REFS       16   DEFINED       13       22
```

```
        PROGRAM EXPDIFF     74/74    OPT=1                              FTN 4.5+414       06/04/76  13.30.36       PAGE     3

 VARIABLES       SN  TYPE            RELOCATION
  4464  F            REAL                              REFS        26        27        31        32        39        40
  4466  FE           REAL                              REFS        26        27        28      2*30        31        37        38
  4465  FT           REAL                              REFS        26        27
  4470  FTE          REAL                              REFS        26        32        40
  4467  FTT          REAL                              REFS        26        32        40
  4476  G            REAL                              REFS        44        45        49        50        57        58
  4500  GE           REAL                              REFS        44        45        46      2*44        49        55        56
  4477  GT           REAL                              REFS        44        45
  4501  GTE          REAL                              REFS        44        50        58
  4454  I            INTEGER                           REFS        24        25        26        28        29      2*31        32
                                                     2*33        34      2*39        40      2*41        42        44        46
                                                       47      2*49        50      2*51        52      2*57        58      2*59
                                                       60        63   DEFINED        12        34        42        52        60
  4503  IK           INTEGER                           REFS        84   DEFINED        82
  4462  J            INTEGER                           REFS        18   DEFINED        13
  4452  JC        *  REAL         *UNDEF               REFS         2
  4461  M            INTEGER                           REFS        64   DEFINED        13
  4502  MAX          INTEGER                           REFS        80        81
     3  NU           REAL                    A         REFS         2         3   DEFINED        10
  4463  N            INTEGER                           REFS        19        20        22        64        80
                                                    DEFINED        63
  4507  NABS         REAL          ARRAY               REFS         2        81   DEFINED        75        76        77
  4453  NE        *  REAL         *UNDEF               REFS         2
  4504  NORD         REAL          ARRAY               REFS         2        81   DEFINED        72        73        74
     2  Q            REAL                    A         REFS         3   DEFINED         9
     0  R            REAL                    A         REFS         3   DEFINED         7
     4  S            REAL                    A         REFS         3   DEFINED         6
  4473  S1           REAL                              REFS        39        57   DEFINED        36        54
  4474  S2           REAL                              REFS        39        57   DEFINED        37        55
  4475  S3           REAL                              REFS        39        57   DEFINED        38        56
     0  T            REAL          ARRAY     B         REFS         4        19        21        24        25        33        41
                                                       51        59        81   DEFINED        15        19        33        41
                                                       51        59
 35230  TERR         REAL          ARRAY     B         REFS         4   DEFINED        17        32        40        50        58
  4512  TITLE        REAL          ARRAY               REFS         5        81   DEFINED        67        68        69        70
                                                       71
  4457  TMAX         REAL                              REFS        24        64   DEFINED        13
  4456  TSTART       REAL                              REFS        15        64   DEFINED        13        21
  4472  X            REAL                              REFS        31        49   DEFINED        30        48

 FILE NAMES          MODE
     0  INPUT
  2041  OUTPUT
     0  TAPE5        FMT                     READS        13        82
  2041  TAPE6

 EXTERNALS             TYPE     ARGS        REFERENCES
        EXP            REAL       1 LIBRARY     30        48
        FCALC                     6             26
        GCALC                     5             44
        DTWRIT                    5             64
        PLOT                      2             80
        PRTPLT                    9             81

 INLINE FUNCTIONS      TYPE     ARGS        DEF LINE  REFERENCES
        ABS            REAL       1 INTRIN              28        46
```

APPENDIX B

```
        PROGRAM EXPDIFF     74/74   OPT=1                          FTN 4.5+414       06/04/76  13.30.36       PAGE     4

STATEMENT LABELS                 DEF LINE    REFERENCES
 4121  10                           11          04
 4412  20       FMT                 14          13
 4136  30                           23          18          35          43          53          61
 4175  40                           36          29
 4223  50                           44          25
 4256  60                           54          47
 4302  70                           62          24
 4422  80       FMT                 83          82
 4337  90                           85          84

COMMON BLOCKS    LENGTH      MEMBERS   BIAS NAME(LENGTH)
        A            5                 0 R       (1)                 1 EPS    (1)               2 Q       (1)
                                       3 MU      (1)                 4 S      (1)
        B        20000                 0 T       (5000)           5000 E      (5000)        10000 DELTA   (5000)
                                   15000 TERR    (5000)

STATISTICS
  PROGRAM LENGTH                     414B      268
  BUFFER LENGTH                     4103B     2115
  CM LABELED COMMON LENGTH         47045B    20005
```

```
SUBROUTINE FCALC       74/74    OPT=1                    FTN 4.5+414        06/04/76  13.30.36        PAGE      1

                 SUBROUTINE FCALC(I,FDFTE,FSUBT,FSUBE,FSUBTT,FSUBTE)
                 REAL JC,MU,NE
                 COMMON/A/R,EPS,Q,MU,S
                 COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
                 RT=R*T(I)
                 FDFTE=(JC(I)-Q*MU*NE(I)*E(I)*EXP(RT))/EPS
                 FSUBTE=(JC(I)-Q*MU*NE(I)*E(I)*EXP(RT))*(R/EPS)
                 FSUBE=-(Q*MU*NE(I)*EXP(RT))/EPS
                 FSUBTT=(JC(I)-Q*MU*NE(I)*E(I)*EXP(RT))*(R**2)/EPS
                 FSUBTE=-(Q*MU*NE(I)*R*EXP(RT))/EPS
                 RETURN
                 END


       SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE      REFERENCES
    3  FCALC          1            11

VARIABLES       SN  TYPE            RELOCATION
23420  DELTA        REAL        ARRAY    B            REFS       4
11610  E            REAL        ARRAY    B            REFS       4      6      7      9
    1  EPS          REAL                 A            REFS       3      6      7      8      9      10
    0  FDFTE        REAL                     F.P.     DEFINED    1      6
    0  FSUBE        REAL                     F.P.     DEFINED    1      8
    0  FSUBT        REAL        *UNUSED      F.P.     DEFINED    1
    0  FSUBTE       REAL                     F.P.     DEFINED    1      7      10
    0  FSUBTT       REAL                     F.P.     DEFINED    1      9
    0  I            INTEGER                  F.P.     REFS       5      3*6    3*7    8      3*9    10
                                                      DEFINED    1
    3  MU           REAL                 A            REFS       2      3      6      7      8      9      10
    2  Q            REAL                 A            REFS       3      6      7      8      9      10
    0  R            REAL                 A            REFS       3      5      7      9      10
  113  RT           REAL                              REFS       6      7      8      9      10
                                                      DEFINED    5
    4  S            REAL                 A            REFS       3
    0  T            REAL        ARRAY    B            REFS       4      5
35230  TERR         REAL        ARRAY    B            REFS       4

EXTERNALS            TYPE    ARGS          REFERENCES
       EXP           REAL      2 LIBRARY       6      7      8      9      10
       JC            REAL      1               2      6      7      9
       NE            REAL      1               2      6      7      8      9      10

COMMON BLOCKS     LENGTH       MEMBERS - BIAS NAME(LENGTH)
     A                5              0 R      (1)              1 EPS    (1)             2 Q      (1)
                                     3 MU     (1)              4 S      (1)
     B            20000              0 T      (5000)        5000 E      (5000)      10000 DELTA  (5000)
                                 15000 TERR   (5000)

STATISTICS
  PROGRAM LENGTH                  114B       76
  CM LABELED COMMON LENGTH     47045B     20005
```

```
      SUBROUTINE GCALC       74/74    OPT=1                      FTN 4.5+414       06/04/76  13.30.36       PAGE     1

   1                 SUBROUTINE GCALC(I,GDFTE,GSUBT,GSUBE,GSUBTE)
                     REAL JC,MU,NE
                     COMMON/A/R,EPS,Q,MU,S
                     COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
   5                 RT=R*T(I)
                     GDFTE=(JC(I)-Q*MU*NE(I)*E(I))/EPS
                     GSUBT=-(Q*MU*E(I)*S)/EPS
                     GSUBE=-(Q*MU*NE(I))/EPS
                     GSUBTE=-(Q*MU*S)/EPS
  10                 RETURN
                     END

      SYMBOLIC REFERENCE MAP (R=3)

 ENTRY POINTS      DEF LINE      REFERENCES
     3  GCALC          1           10

 VARIABLES      SN  TYPE           RELOCATION
 23420  DELTA       REAL       ARRAY   B          REFS       4
 11610  E           REAL       ARRAY   B          REFS       4        6        7
     1  EPS         REAL               A          REFS       3        6        7        8        9
     0  GDFTE       REAL                  F.P.   DEFINED     1        6
     0  GSUBE       REAL                  F.P.   DEFINED     1        8
     0  GSUBT       REAL                  F.P.   DEFINED     1        7
     0  GSUBTE      REAL                  F.P.   DEFINED     1        9
     0  I           INTEGER               F.P.    REFS       5       3*6       7        8    DEFINED      1
     3  MU          REAL               A          REFS       2        3        6        7        8        9
     2  Q           REAL               A          REFS       3        6        7        8        9
     0  R           REAL               A          REFS       3        5
    40  RT        * REAL                         DEFINED     5
     4  S           REAL               A          REFS       3        7        9
     0  T           REAL       ARRAY   B          REFS       4        5
 35230  TERR        REAL       ARRAY   B          REFS       4

 EXTERNALS            TYPE    ARGS        REFERENCES
        JC            REAL     1               2         6
        NE            REAL     1               2         6         8

 COMMON BLOCKS    LENGTH      MEMBERS - BIAS NAME(LENGTH)
        A            5                  0 R      (1)               1 EPS     (1)               2 Q       (1)
                                        3 MU     (1)               4 S       (1)
        B        20000                  0 T      (5000)         5000 E       (5000)        10000 DELTA   (5000)
                                    15000 TERR   (5000)

 STATISTICS
   PROGRAM LENGTH                    41B       33
   CM LABELED COMMON LENGTH      47045B     20005
```

```
SUBROUTINE OTWRIT      74/74    OPT=1                          FTN 4.5+414        06/04/76  13.30.36        PAGE     1

              SUBROUTINE OTWRIT(N,M,DEL,TSTART,TMAX)
              COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
              IT=0
              K=1
              WRITE(6,10)
           10 FORMAT(1H1)
              WRITE(6,20)DEL,TSTART,TMAX,M
           20 FORMAT(1X,17HDELTA TIME STEP: ,1PE10.3,5X,15HSTARTING TIME: ,
             11PE10.3,5X,20HMAXIMUM TIME TO BE PLOTTED: ,1PE10.3,/,34X,
             220HINCREMENT OF OUTPUT VALUES: ,I5,/,/,/,/)
              WRITE(6,30)
           30 FORMAT(4X,4HTIME,14X,1HE,7X,16HTRUNCATION ERROR,8X,5HDELTA)
              WRITE(6,40)
           40 FORMAT(4X,4H----,14X,1H-,7X,16H----------------,8X,5H-----)
              DO 110 I=1,N,M
              WRITE(6,50)T(I),E(I),TERR(I),DELTA(I)
           50 FORMAT(1X,4(1PE11.4,5X))
              IF(IT.EQ.1)  GO TO 60
              IF(K.EQ.50)  GO TO 70
           60 IF(K.EQ.60)  GO TO 70
              K=K+1
              GO TO 110
           70 CONTINUE
              WRITE(6,80)
           80 FORMAT(1H1)
              WRITE(6,90)
           90 FORMAT(4X,4HTIME,14X,1HE,7X,16HTRUNCATION ERROR,8X,5HDELTA)
              WRITE(6,100)
          100 FORMAT(4X,4H----,14X,1H-,7X,16H----------------,8X,5H-----)
              IT=1
              K=1
          110 CONTINUE
              RETURN
              END

    SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS      DEF LINE      REFERENCES
    3  OTWRIT         1             33

VARIABLES     SN  TYPE           RELOCATION
    0  DEL        REAL                F.P.        REFS        7   DEFINED      1
23420  DELTA      REAL       ARRAY    B           REFS        2      16
11610  E          REAL       ARRAY    B           REFS        2      16
  206  I          INTEGER                         REFS     4*16   DEFINED     15
  204  IT         INTEGER                         REFS       18   DEFINED      3        30
  205  K          INTEGER                         REFS       19      20       21   DEFINED      4      21      31
    0  M          INTEGER             F.P.        REFS        7      15  DEFINED       1
    0  N          INTEGER             F.P.        REFS       15   DEFINED      1
    0  T          REAL       ARRAY    B           REFS        2      16
35230  TERR       REAL       ARRAY    B           REFS        2      16
    0  TMAX       REAL                F.P.        REFS        7   DEFINED      1
    0  TSTART     REAL                F.P.        REFS        7   DEFINED      1
```

```
      SUBROUTINE OTWRIT      74/74   OPT=1                      FTN 4.5+414      06/04/76  13.30.36      PAGE    2

   FILE NAMES        MODE
         TAPE6      FMT                   WRITES       5       7       11      13      16      24      26      28

   STATEMENT LABELS               DEF LINE    REFERENCES
       70  10          FMT              6           5
      101  20          FMT              8           7
      124  30          FMT             12          11
      135  40          FMT             14          13
      152  50          FMT             17          16
       64  60                          20          18
       50  70                          23          19         20
      160  80          FMT             25          24
      165  90          FMT             27          26
      176  100         FMT             29          28
       60  110                         32          15         22

   LOOPS  LABEL     INDEX      FROM-TO     LENGTH     PROPERTIES
      30  110     * I          15 32       33B                    EXT REFS

   COMMON BLOCKS    LENGTH      MEMBERS - BIAS NAME(LENGTH)
         B          20000                 0 T         (5000)          5000 E      (5000)        10000 DELTA  (5000)
                                      15000 TERR      (5000)

   STATISTICS
     PROGRAM LENGTH                2218      145
     CM LABELED COMMON LENGTH    47040B    20000
```

```
SUBROUTINE PLOT       74/74    OPT=1                              FTN 4.5+414        06/04/76  13.30.36        PAGE     1

                      SUBROUTINE PLOT(N,MAX)
                      COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
                      RT=N/100.
                      IDEL=INT(RT)
                      IF(IDEL.EQ.0)  IDEL=1
                      L=1
                      DO 10 I=1,N,IDEL
                      T(L)=T(I)
                      E(L)=E(I)
                      L=L+1
                   10 CONTINUE
                      T(L)=T(N)
                      E(L)=E(N)
                      MAX=L
                      RETURN
                      END


      SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS     DEF LINE      REFERENCES
    3 PLOT           1           15

VARIABLES      SN  TYPE             RELOCATION
23420  DELTA       REAL        ARRAY   B            REFS        2
11610  E           REAL        ARRAY   B            REFS        2        9        13     DEFINED       9        13
   37  I           INTEGER                          REFS        8        9    DEFINED       7
   35  IDEL        INTEGER                          REFS        5        7    DEFINED       4        5
   36  L           INTEGER                          REFS        8        9        10       12       13       14
                                                 DEFINED        6       10
    0  MAX         INTEGER               F.P.    DEFINED        1       14
    0  N           INTEGER               F.P.       REFS        3        7        12       13    DEFINED       1
   34  RT          REAL                             REFS        4   DEFINED        3
    0  T           REAL        ARRAY   B            REFS        2        8        12     DEFINED       8        12
35230  TERR        REAL        ARRAY   B            REFS        2

INLINE FUNCTIONS    TYPE    ARGS         DEF LINE   REFERENCES
     INT         INTEGER     1  INTRIN                  4

STATEMENT LABELS             DEF LINE    REFERENCES
    0  10                       11           7

LOOPS  LABEL     INDEX     FROM-TO     LENGTH      PROPERTIES
   17  10          I          7 11        4B        INSTACK

COMMON BLOCKS    LENGTH     MEMBERS - BIAS NAME(LENGTH)
    B            20000                0 T        (5000)           5000 E       (5000)            10000 DELTA  (5000)
                                  15000 TERR     (5000)

STATISTICS
  PROGRAM LENGTH              40B        32
  CM LABELED COMMON LENGTH    47040B     20000
```

APPENDIX B

```
      FUNCTION NE         74/74    OPT=1                         FTN 4.5+414        06/04/76  13.30.36        PAGE     1

    1                  REAL FUNCTION NE(I)
                       REAL NEO
                       COMMON/A/R,EPS,Q,MU,S
                       COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
    5                  NEO=1.133E+11
                       IF(T(I).GE.0.)  GO TO 10
                C  NE MULTIPLIED BY 1.E+6 FOR CORRECTION TO METERS
                       NE=NEO*1.E+6
                       RETURN
   10           C  NE MULTIPLIED BY 1.E+6 FOR CORRECTION TO METERS
                  10   NE=(NEO+S*T(I))*1.E+6
                       RETURN
                       END


     SYMBOLIC REFERENCE MAP (R=3)

 ENTRY POINTS      DEF LINE      REFERENCES
     4  NE             1            9        12

 VARIABLES     SN  TYPE            RELOCATION
 23420  DELTA      REAL      ARRAY   B            REFS        4
 11610  E          REAL      ARRAY   B            REFS        4
     1  EPS        REAL              A            REFS        3
     0  I          INTEGER             F.P.       REFS        6        11    DEFINED       1
     3  MU         INTEGER           A            REFS        3
    22  NE         REAL                        DEFINED        8        11
    23  NEO        REAL                           REFS        2         8          11    DEFINED       5
     2  Q          REAL              A            REFS        3
     0  R          REAL              A            REFS        3
     4  S          REAL              A            REFS        3        11
     0  T          REAL      ARRAY   B            REFS        4         6          11
 35230  TERR       REAL      ARRAY   B            REFS        4

 STATEMENT LABELS             DEF LINE    REFERENCES
    13  10                        11           6

 COMMON BLOCKS    LENGTH      MEMBERS - BIAS NAME(LENGTH)
       A              5                0 R      (1)             1 EPS     (1)             2 Q       (1)
                                       3 MU     (1)             4 S       (1)
       B          20000                0 T      (5000)       5000 E       (5000)      10000 DELTA   (5000)
                                   15000 TERR   (5000)

 STATISTICS
   PROGRAM LENGTH                 24B        20
   CM LABELED COMMON LENGTH    47045B     20005
```

```
FUNCTION JC          74/74   OPT=1                          FTN 4.5+414        06/04/76  13.30.36        PAGE     1

1                    REAL FUNCTION JC(I)
                     COMMON/A/R,EPS,Q,MU,S
                     COMMON/B/T(5000),E(5000),DELTA(5000),TERR(5000)
                     IF(T(I).GE.0.)  GO TO 10
5                    JC=1.34E+3*EXP(R*T(I))
                     RETURN
                  10 JC=1.34E+3
                     RETURN
                     END


      SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS     DEF LINE      REFERENCES
    4  JC            1              6         8

VARIABLES      SN  TYPE          RELOCATION
23420  DELTA       REAL       ARRAY   B         REFS        3
11610  E           REAL       ARRAY   B         REFS        3
    1  EPS         REAL               A         REFS        2
    0  I           INTEGER               F.P.   REFS        4          5    DEFINED        1
   17  JC          REAL                      DEFINED        5          7
    3  MU          INTEGER            A         REFS        2
    2  Q           REAL               A         REFS        2
    0  R           REAL               A         REFS        2          5
    4  S           REAL               A         REFS        2
    0  T           REAL       ARRAY   B         REFS        3          4         5
35230  TERR        REAL       ARRAY   B         REFS        3

EXTERNALS          TYPE    ARGS         REFERENCES
       EXP         REAL      1 LIBRARY       5

STATEMENT LABELS            DEF LINE    REFERENCES
   14  10                      7            4

COMMON BLOCKS   LENGTH     MEMBERS - BIAS NAME(LENGTH)
      A             5               0 R      (1)               1 EPS     (1)                 2 Q      (1)
                                    3 MU     (1)               4 S       (1)
      B         20000               0 T      (5000)         5000 E       (5000)          10000 DELTA  (5000)
                                15000 TERR   (5000)

STATISTICS
  PROGRAM LENGTH                   20B       16
  CM LABELED COMMON LENGTH      47045B    20005
```

APPENDIX B

```
SUBROUTINE PRTPLT     74/74    OPT=1                    FTN 4.6+420         10/27/76  09.31.03        PAGE     1

                 SUBROUTINE PRTPLT (HORZ,VERT,NPTS,HEAD,HLAB,VLAB,HREF,VREF,J)
            C
            C    THIS FORTRAN SUBROUTINE PRODUCES A FULL PAGE X-Y PRINTER PLOT.
            C
            C    HORZ - REAL ARRAY TO BE PLOTTED ALONG HORIZONTAL AXIS.
            C    VERT - REAL ARRAY TO BE PLOTTED ALONG VERTICAL AXIS.
            C    NPTS - NUMBER OF VALUES IN HORZ,VERT (NTE 101)
            C    HEAD - PLOT HEADING (MUST BE 5 WORDS)
            C    HLAB - LABEL FOR HORIZONTAL AXIS (MUST BE 3 WORDS)
            C    VLAB - LABEL FOR VERTICAL AXIS (MUST BE 3 WORDS)
            C    HREF - REFERENCE VALUE FOR HORIZONTAL AXIS.
            C    VREF - REFERENCE VALUE FOR VERTICAL AXIS.
            C    J  THIS DETERMINES THE SIZE OF THE ARRAYS HORZ AND VERT--NTE 100
            C
                 DIMENSION HORZ(J),VERT(J),HEAD(5),HLAB(3),VLAB(3),LINE(101),
                C IVER(101),JHOR(101),HORIZ(101),VERTIC(101),YAXIS(30),DOWNT(21)
                 INTEGER BLANK,STAR,DOT,EYE
            C
            C
                 DATA BLANK/1H /,STAR/1H*/,DOT/1H./,MINUS/1H-/,EYE/1HI/
                 CALL DATE(DAT)
                 CALL TIME(TIM)
                 HEAD(4)=DAT
                 HEAD(5)=TIM
                 NO = 1
                 NUM = 1
                 N = NPTS
                 N = MINO(N,101)
                 DO 10 I = 1,N
                 HORIZ(I) = HORZ(I)
                 VERTIC(I) = VERT(I)
              10 CONTINUE
            C
                 DECODE (30,1000,VLAB(1)) (YAXIS(I),I=1,30)
            1000 FORMAT(30A1)
              20 NA = N - 1
                 DO 30 I = 1,NA
                 IF (VERTIC(I).GE.VERTIC(I+1)) GO TO 30
                 HOLD = VERTIC(I)
                 VERTIC(I) = VERTIC(I+1)
                 VERTIC(I+1) = HOLD
                 SAVE = HORIZ(I)
                 HORIZ(I) = HORIZ(I+1)
                 HORIZ(I+1) = SAVE
              30 CONTINUE
                 DO 40 I = 1,NA
                 IF (VERTIC(I).LT.VERTIC(I+1)) GO TO 20
              40 CONTINUE
                 VMAX = VERTIC(1)
                 VMIN = VERTIC(N)
                 HMAX = HORIZ(1)
                 HMIN = HORIZ(1)
                 DO 50 I = 2,N
```

```
SUBROUTINE PRTPLT     74/74    OPT=1                    FTN 4.6+420        10/27/76  09.31.03        PAGE     2

            HZ = HORIZ(I)
            HMAX = AMAX1(HMAX,HZ)
            HMIN = AMIN1(HMIN,HZ)
         50 CONTINUE
            DO 60 I = 1,K
            JVER = 50.*(VERTIC(I)-VMIN)/(VMAX-VMIN) + 1.5
            IVER(I) = 52 - JVER
         60 JHOR(I) = 100.*(HORIZ(I)-HMIN)/(HMAX-HMIN) + 1.5
            KHREF = 100.*(HREF-HMIN)/(HMAX-HMIN) + 1.5
            IF (KHREF.LT.1) KHREF = 150
            IF (KHREF.GT.101) KHREF = 150
            KVREF = 50.*(VREF-VMIN)/(VMAX-VMIN) + 1.5
            KVREF = 52 - KVREF
            IF (KVREF.LT.1) KVREF = 1
            IF (KVREF.GT.51) KVREF = 1
            UP = (VMAX-VMIN)/50.
            PRINT 2000 , HEAD
       2000 FORMAT(1H1,35X,5A10//)
            DO 200 K = 1,51
            LINE(1) = EYE
            LINE(101) = EYE
            DO 70 I = 2,100
         70 LINE(I) = BLANK
            IF (K.EQ.1) GO TO 90
            IF (K.EQ.51) GO TO 90
            IF (K.NE.KVREF) GO TO 120
            DO 80 I = 2,100
         80 LINE(I) = DOT
            GO TO 120
         90 DO 100 I = 2,100
        100 LINE(I) = MINUS
            DO 110 I = 6,100,5
        110 LINE(I) = EYE
            GO TO 130
        120 IF (KHREF.EQ.150) GO TO 130
            IF (KHREF.EQ.1) GO TO 130
            LINE(KHREF) = DOT
        130 IF (K.EQ.IVER(NUM)) GO TO 140
            IF (K.LT.IVER(NUM)) GO TO 160
            NUM = NUM + 1
            GO TO 130
        140 IF (JHOR(NUM).GT.101) GO TO 150
            IF (JHOR(NUM).LT.1) GO TO 150
            JH = JHOR(NUM)
            LINE(JH) = STAR
        150 NUMBER = NUM + 1
            IF (IVER(NUM).NE.IVER(NUMBER)) GO TO 160
            NUM = NUMBER
            GO TO 140
        160 T = K - 1
            UPT = VMAX - T*UP
            IF (K.LT.11) GO TO 170
            IF (K.GT.41) GO TO 170
```

```
SUBROUTINE PRTPLT     74/74    OPT=1                    FTN 4.6+420       10/27/76  09.31.03       PAGE     3

              TAXIS = VAXIS(NO)
              NO = NO + 1
              PRINT 3000 , TAXIS,UPT,LINE
         3000 FORMAT(4X,A1,4X,1PE9.2,2X,101A1)
              GO TO 200
          170 PRINT 4000 , UPT,LINE
         4000 FORMAT(9X,1PE9.2,2X,101A1)
          200 CONTINUE
              DOWN = (HMAX-HMIN)/20.
              T = HMIN
              DO 210 I = 1,21
              DOWNT(I) = T
              T = T + DOWN
          210 CONTINUE
              PRINT 5000 , (DOWNT(I),I=1,21,2)
         5000 FORMAT(/15X,1P11E10.2)
              PRINT 6000 , (DOWNT(I),I=2,20,2)
         6000 FORMAT(20X,1P10E10.2)
              PRINT 7000 , HLAB
         7000 FORMAT(/50X,3A10)
              RETURN
              END
```

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